11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

EFFICIENT PARALLEL SOLUTION METHODS FOR MORTAR FINITE ELEMENT DISCRETIZATIONS IN COMPUTATIONAL CONTACT MECHANICS

Alexander Popp*, **Philipp Farah**, **Tobias Wiesner** and **Wolfgang A. Wall**

Institute for Computational Mechanics, Technische Universität München,
Boltzmannstr. 15, D-85748 Garching b. München, Germany,
popp@lnm.mw.tum.de, http://www.lnm.mw.tum.de

Key words: *Mortar Finite Element Methods, Finite Deformation Contact, Parallel Solution Methods, Numerical Integration, Iterative Solvers, Algebraic Multigrid.*

The past decade has seen a great thrust of research activities on new robust discretization schemes and efficient solution algorithms for computational contact mechanics. Mortar finite element discretizations are now commonly accepted as state-of-the-art approach with significant advantages over traditional methods (e.g. node-to-segment approach) in terms of accuracy and robustness, especially in the context of nonlinear kinematics with finite deformations and finite (frictional) sliding. Both standard [2] and so-called dual mortar methods based on biorthogonal Lagrange multiplier bases [4] have reached a remarkable level of maturity, with the latter variant arguably offering additional benefits, such as simple condensation procedures for the discrete Lagrange multipliers and a natural compatibility with efficient nonlinear solution schemes using nonlinear complementarity (NCP) functions and semi-smooth Newton methods, see e.g. [3].

Thus, many researchers, including the authors of this contribution, have started looking into special topics for mortar-based contact discretizations, for example the modeling of associated physical effects (e.g. wear, adhesion or thermomechanical coupling) or the extension towards smooth interpolation methods (e.g. isogeometric analysis). While these are all worthwhile research directions, we want to address another, at least equally important aspect in this contribution: the development of efficient parallel solution methods for such mortar-based contact discretizations, so that they become applicable to complex large-scale contact problems in industrial practice. Specifically, our recent advances with regard to the following three topics will be outlined: (i) scalable algorithms based on parallel redistribution and dynamic load balancing, (ii) efficient numerical integration schemes for mortar methods and (iii) novel iterative linear solvers / preconditioners for mortar-based contact using algebraic multigrid (AMG) techniques. All developments are carried out within our multi-physics research code BACI, a parallel simulation environment that makes use of overlapping domain decomposition and MPI communication.

With regard to scalable algorithms, a parallel redistribution of the contact interfaces independent of the underlying bulk discretization and a dynamic load balancing strategy specifically developed for finite deformations are investigated. We demonstrate that the resulting algorithms indeed assure optimal scalability of mortar-based contact evaluation over a wide range of elements per core within a common multi-core architecture.

In terms of efficient numerical integration, we revisit the well-known fact that the discrete mortar coupling terms at contact interfaces exhibit discontinuities (kinks) due to non-matching slave and master surface meshes and thus require special treatment. Basically, two different types of numerical integration schemes, denoted as *segment-based* integration and *element-based* integration here, can be found predominantly in the literature. While the entire existing literature focuses on either of the two mentioned approaches without questioning this choice, the intention here is to provide a comprehensive and unbiased comparison. The covered aspects include the choice of integration rule, the treatment of boundaries of the contact zone, higher-order interpolation and frictional sliding. Moreover, a new hybrid scheme is proposed, which beneficially combines the advantages of segment-based and element-based mortar integration [1].

Finally, we present a new class of iterative linear solvers and preconditioners based on AMG methods, which is specifically tailored for linear systems arising from mortar-based discretizations (especially *dual* mortar-based discretizations) in contact mechanics. An out-of-the-box application of highly efficient AMG methods developed for unconstrained solid mechanics problems may yield satisfactory results in some cases, but the non-conforming structure of mortar-based discretizations precludes an efficient solution in general. In order to tap the full potential of such techniques, we aim at including available knowledge about the physics and numerics of the underlying contact problem in the solver design. The main features of our new AMG methods for mortar-based contact are a permutation strategy that improves diagonal dominance and an aggregation strategy that avoids coarse level aggregates overlapping the contact interface.

REFERENCES

[1] P. Farah, A. Popp and W.A. Wall. Segment-based vs. element-based integration for mortar methods in computational contact mechanics. *Submitted for publication*, 2013.

[2] T.A. Laursen, M.A. Puso and J. Sanders. Mortar contact formulations for deformable-deformable contact: past contributions and new extensions for enriched and embedded interface formulations. *Computer Methods in Applied Mechanics and Engineering*, Vol. **205–208**, 3–15, 2012.

[3] A. Popp, A. Seitz, M.W. Gee and W.A. Wall. Improved robustness and consistency of 3D contact algorithms based on a dual mortar approach. *Computer Methods in Applied Mechanics and Engineering*, Vol. **264**, 67–80, 2013.

[4] B.I. Wohlmuth. Variationally consistent discretization schemes and numerical algorithms for contact problems. *Acta Numerica*, Vol. **20**, 569–734, 2011.