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THERMOELASTICITY IN FGM SHELL STRUCTURES

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An efficient low order shell formulation is presented for analyzing thermo-elastic effects in shell structures made of **F**unctionally **G**raded **M**aterials (FGM), where material properties show an arbitrary continuous distribution. The discretization of the mid-surface of a curved shell geometry of thickness h leads to possibly warped element geometries. In case of non-symmetric variations of Young's modulus E with respect to the geometric mid-plane ($\hat{z} = 0$), where $\hat{z}$ denotes the thickness direction, the corresponding nodes are projected to a mechanical neutral plane, where bending and membrane properties decouple. The offset of this neutral plane from the geometrical mid-surface reads

$$\overline{z} = \frac{1}{\int_{-h/2}^{h/2} E(\hat{z}) d\hat{z}} \int_{-h/2}^{h/2} E(\hat{z}) \hat{z} d\hat{z}. \tag{1}$$

From this warped element configuration we extract a plane element configuration as discussed in [1] in order to derive a plate element independent from a membrane bending element, where the frequently missing drilling rotations are included based on a recently proposed functional [2]. The details of the corresponding element formulation is given in [1] where effective elastic quantities are introduced. The effective moduli for membrane and bending read

$$E_m = \frac{1}{h} \int_{-h/2-\overline{z}}^{h/2-\overline{z}} E(\hat{z}') d\hat{z}' \ , \quad E_b = \frac{12}{h^3} \int_{-h/2-\overline{z}}^{h/2-\overline{z}} E(\hat{z}') \hat{z}'^2 d\hat{z}', \tag{2}$$

while the shear correction factor is

$$\alpha_s = \left(\frac{144}{E_b h^5} \int_{-h/2-\overline{z}}^{h/2-\overline{z}} \frac{1}{E(\hat{z}')} \left[\int_{\hat{z}'}^{h/2-\overline{z}} E(\zeta) \zeta d\zeta \right]^2 d\hat{z}' \right)^{-1}. \tag{3}$$

The weak one way coupling to introduce thermo-elastic effects is based on the virtual work principle which reads in the absence of mechanical loads $\delta\Pi = \int (\delta\boldsymbol{\varepsilon})^T \boldsymbol{\sigma} dV = 0$.

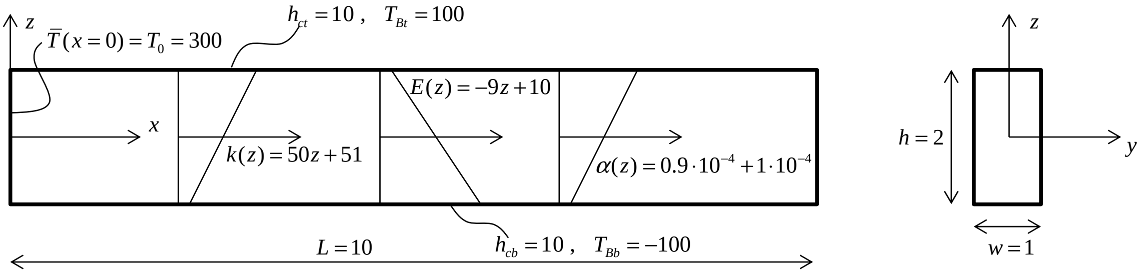

Figure 1: FGM thermo-elastic fin

Here δ denotes the variation symbol, the strain field omitting the shear terms[1] reads $\boldsymbol{\varepsilon} = [\varepsilon_{\hat{x}\hat{x}}\, \varepsilon_{\hat{y}\hat{y}}]^T$ while the corresponding stresses are $\boldsymbol{\sigma} = [\sigma_{\hat{x}\hat{x}}\, \sigma_{\hat{y}\hat{y}}]^T$. Introducing a thermo-elastic constitutive equation for isotropic materials, i.e.

$$\begin{bmatrix} \sigma_{\hat{x}\hat{x}} \\ \sigma_{\hat{y}\hat{y}} \end{bmatrix} = \frac{E(\hat{z})}{1-\nu^2} \begin{bmatrix} 1 & \nu \\ \nu & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{\hat{x}\hat{x}} \\ \varepsilon_{\hat{y}\hat{y}} \end{bmatrix} - \frac{E(\hat{z})\alpha(\hat{z})\Delta T(\hat{z})}{1-\nu} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \tag{4}$$

with $\alpha(\hat{z})$ referring to the thermal expansion coefficient and $\Delta T(\hat{z})$ denoting temperature elevations which may depend on the thickness coordinate. Introducing (4) into the virtual work term gives $\int (\delta\boldsymbol{\varepsilon})^T \frac{E(\hat{z})}{1-\nu^2} \begin{bmatrix} 1 & \nu \\ \nu & 1 \end{bmatrix} \boldsymbol{\varepsilon} dV - \int (\delta\boldsymbol{\varepsilon})^T \frac{E(\hat{z})\alpha(\hat{z})\Delta T(\hat{z})}{1-\nu} \begin{bmatrix} 1 \\ 1 \end{bmatrix} dV$, where the left hand side leads to the finite element formulation discussed in [1], while the right hand side refers to internal forces and couples caused by thermal expansion. Those internal forces and couples are proportional to $f = \int_{-h/2}^{h/2} \frac{E(\hat{z})\alpha(\hat{z})\Delta T(\hat{z})}{1-\nu} d\hat{z}$ and $m = \int_{-h/2}^{h/2} \hat{z} \frac{E(\hat{z})\alpha(\hat{z})\Delta T(\hat{z})}{1-\nu} d\hat{z}$, while the integration with respect to the membrane directions can be evaluated analytically. Consider a fin with a rectangular cross section made of a FGM with linear distributions of all relevant properties according to Fig. 1. The fin is loaded with a mean temperature at $x = 0$ while convection is applied on the top and bottom surface with corresponding convection coefficients h_{ct} and h_{cb} and fluid temperatures of T_{Bt} and T_{Bb}. A suitable procedure to calculate the temperature field is discussed in [3]. The evaluated displacements at $x = L$ are compared to an ANSYS continuum solution indicating good accuracy of the proposed algorithm ($u_x = 0.0987$ error: 1.5%; $u_z = -0.6782$ error: 0.5%).

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[1] The shear terms may be omitted since temperature elevations cause only normal strains.