11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

On the Application of the Method of Difference Potentials to Linear Elastic Fracture Mechanics

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Key Words: *method of difference potentials, fracture, extended finite element method, rate of convergence.*

The Difference Potential Method (DPM) [1] proved to be a very efficient tool for solving boundary value problems (BVPs) in complex regions. It allows BVPs to be reduced to a boundary equation without the knowledge of Green's functions. The method has been successfully used for solving very different problems related to the solution of partial differential equations [1]. However, it has mostly been considered in regular (Lipschitz) domains. In the current paper, for the first time the method has been applied to a problem of linear elastic fracture mechanics. This problem requires solving BVPs in domains with a cut. In DPM the reduction of the BVP to a boundary equation is based on generalized surface projections [1, 2]. The projection is fully determined by the clear trace (notion introduced by Ryaben'kii). In the current paper, for the first time the minimal clear trace found in [3] for the problem with cuts has been numerically realized.

Simple fracture test cases are solved to demonstrate the capability of the method in solving problems containing discontinuities and singularities. As is the case for finite element formulations, the accuracy is found to be compromised by the presence of the singularity at the crack tip. Singular enrichment functions such as those used within Extended Finite Element formulations are used in an effort to account for the singularity [4]. Improved convergence rates are achieved with enrichment and are compared to those achieved by the Extended Finite Element Method. Possible future applications of the method within the field of fracture mechanics are discussed.

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