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STREAMLINE UPWIND FORMULATIONS FOR CALCULATION OF FREE SURFACE CORRECTIONS AND SIMULATION OF STEADY-STATE FORMING PROBLEMS

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The 3D finite element simulations of multi-pass rolling or drawing often result into exorbitant computational times (several weeks or months) that make the numerical approach almost infeasible even by appealing to the best parallel computers. As manufactured products are very long only the “stationary” part of the process is actually of interest for the industry, so the process can be advantageously simulated by resorting to steady-state formulations [1-4] which allows reducing the calculation time by, at least, an order of magnitude with respect to more conventional methods [5]. Literature on steady-state formulations is rather scarce and old, except related works on multi-fields formulations [6] over a prescribed geometry. Most advanced results have been achieved using structured hexahedral meshes. They make it possible to more easily integrate differential equations along the streamlines, including displacement fields that give material geometry. Main methods are based on an iterative free surface algorithm that alternatively computes the flow for a given geometry and then the correction of the geometry for a given flow. Direct problem resolution requires using multi-fields formulations by resorting, for instance, to a reference frame formulation [7]. Contact inequalities can be considered but literature only refers to very simple configurations [7].

This work is carried out under the constraints of using unstructured meshes based on tetrahedrons, of allowing parallel computations on partitioned domains, of handling complex geometries and processes that are very sensitive to contact equations. Therefore, within a velocity / pressure formulation of the metal forming problem, the steady-state problem equations are solved by an iterative approach. It consists in alternating computations of the velocity field with corrections of the domain surface. The steady flow is easily computed and a Streamline Upwing Petrov Galerkin (SUPG) formulation is used to integrate state variables along the streamlines. Our contribution focuses on the computation of the correction $\boldsymbol{t}$ to the current surface geometry $\boldsymbol{X}$ (1) that makes it possible to satisfy the free surface equation (2a) and the contact inequality (2b)

$$\boldsymbol{x} = \boldsymbol{X} + \boldsymbol{t} \tag{1}$$

$$\forall \boldsymbol{x} \in \partial\Omega_{free}, \quad \boldsymbol{v}(\boldsymbol{x})\cdot\boldsymbol{n}(\boldsymbol{x}) = 0 \quad \text{and} \quad \forall \boldsymbol{x} \in \partial\Omega_{contact}, \quad \boldsymbol{t}\cdot\boldsymbol{n}(\boldsymbol{X}) - \delta(\boldsymbol{X}) \le 0 \tag{2}$$

where $\boldsymbol{v}$ is the velocity field, $\boldsymbol{n}$ the surface normal and $\boldsymbol{\delta}$ the contact distance. Different weak formulations (3) are investigated to solve these equations (2) because the standard Galerkin approach (4a) used in [3] provides undetermined solutions in many circumstances. The introduction of the SUPG upwind shift (4b) or the use of a least square formulation (4c) provides much robust formulations. However, the study of a large family of analytic problems shows the necessity to introduce an upwind shift as well in the least square formulation as presented in equations (5) and (6).

$$\forall \varphi^*, \quad \int_{\partial\Omega} \varphi^* \left(\boldsymbol{v} \cdot \boldsymbol{n}(\boldsymbol{t}) \right) dS = 0 \tag{3}$$

$$\varphi^*_{Galerkin} = N \quad ; \quad \varphi^*_{SUPG} = N + \frac{h}{2\|\boldsymbol{v}\|} \nabla \boldsymbol{N} . \boldsymbol{v}_{\text{tangent}} \quad ; \quad \varphi^*_{LS} = \boldsymbol{v} \cdot \frac{\partial \boldsymbol{n}}{\partial \boldsymbol{t}} \tag{4}$$

$$\varphi^*_{LS\_SUPG} = \left(\boldsymbol{1} + \alpha \boldsymbol{C} \right) \boldsymbol{v} \cdot \frac{\partial \boldsymbol{n}}{\partial \boldsymbol{t}} \quad \text{with} \quad \forall k \in \wp_f, \quad C_k = \cos\left(\boldsymbol{X_f X_k}, \boldsymbol{v}_k \right) \quad \text{and} \quad \alpha \in [0, 1] \tag{5}$$

$$\varphi^*_{LS\_upwind} = [\boldsymbol{C}]^+ \boldsymbol{v} \cdot \frac{\partial \boldsymbol{n}}{\partial \boldsymbol{t}} \quad \text{with} \quad [\boldsymbol{C}]^+ = \frac{\boldsymbol{C} + |\boldsymbol{C}|}{2} \quad \text{and} \quad \boldsymbol{C} \in [-1, 1] \tag{6}$$

where $\wp_f$ is the set of surrounding facets to current node *k*. The introduction of the unilateral contact equations (2b) and the application to actual metal forming problems requires introducing a more dramatic upwind shift in the equations as proposed in (6). All these ingredients are tested and validated over a wide range of analytical solutions before being applied to a wider range of actual 3D metal forming applications such as rolling and drawing. The formulation shows remarkably stable and robust. Convergence of the iterative algorithm is reached after about only ten iterations, so providing a very efficient method, which is more than ten times faster than the reference incremental computation of the steady-state.

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