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SECOND ORDER PURE LAGRANGE-GALERKIN METHODS FOR FLUID-STRUCTURE INTERACTION

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We present a second order pure Lagrangian method combined with finite element approximations for the numerical solution of fluid-structure interaction problems. Let us consider the coupling of a viscous Newtonian incompressible fluid and a linear elastic solid. The governing equations for the fluid are the unsteady incompressible Navier-Stokes equations and for the solid the Navier-Lamé equations of linear elasticity. The two equations are coupled by the standard kinematic and kinetic interface conditions, namely, continuity of displacements and forces across the interface. Then both equations are written in material coordinates and in terms of displacements and fluid pressure. A weak formulation for the coupled problem is built where the interface kinematic condition is essential (it is included in the definition of the functional space where the solution is looked for) and the interface kinetic condition is natural. Next we consider a second order time discretization scheme combined with finite element methods for solving the coupled weak formulation: continuous piecewise linear functions for each component of the displacement field in the solid and, in the fluid, the same approximation for the pressure and the mini-element (P_1-bubble) for the displacement field.

We notice that pure Lagrangian displacement methods are useful for solving fluid-structure interaction problems and free surface problems because the computational domain is time independent and the fluid-solid coupling at the interphase is straightforward. Unfortunately, for moderate to high-Reynolds number flows, they can lead to high distortion in the mesh elements. When this happens re-meshing is needed to re-initialize the transformation to the identity.

The proposed methods for time discretization can be considered as methods of characteristics. These methods are based on time discretization of the material time derivative and were introduced in the beginning of the eighties of the last century combined with finite differences or finite elements for space discretization (see [1], [2]). When combined with finite elements they are also called Lagrange-Galerkin methods. For scalar linear convection-diffusion equations, second order pure Lagrange-Galerkin schemes have been introduced in [3] and [4] where stability and error estimates for time semi-discretized and full-discretized schemes have been proved. In [5] a unified formulation to state pure Lagrangian and semi-Lagrangian methods for solving scalar linear convection-diffusion partial differential equations is introduced. More precisely, a quite general change of variable from the current configuration to a reference configuration is proposed obtaining a new strong formulation of the problem from which classical and new time discretization methods can be introduced in a natural way.

When applied to the Navier-Stokes equations, the above ideas lead to displacement methods similar to those used for numerical solution of solid mechanics problems. In [6], a unified formulation to introduce Lagrangian and semi-Lagrangian velocity and displacement methods for solving the Navier-Stokes equations is proposed.

In order to assess the performance of the proposed numerical methods for fluid-structure interaction problems, we solve different problems in two space dimensions. In particular, we present numerical results for a sloshing problem in a rectangular tank with an elastic submerged ball and the fall of an elastic circular disk in a rectangular cavity filled with an incompressible viscous fluid.

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