

# HIGH ORDER DISCONTINUOUS FINITE-VOLUME/FINITE-ELEMENT METHOD FOR CFD APPLICATIONS

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In the framework of computational fluid dynamics (CFD) in engineering applications, low-order methods have been typically considered as the right choice due to their simplicity, robustness, and their effectiveness in providing a reasonably accurate solution by comparably low computational cost. Indeed the majority of the production and commercial codes are first or second order accurate. However, during the past two decades the interest in high order methods has grown not only among the research community, but also in the field of engineering, especially in certain applications where the complex flow structure and small length scales need to be adequately resolved. For instance, in wave propagation problems and in vortex dominated flows, the use of low-order methods may result in unacceptable solutions. In response to this trend, some high order algorithms have been proposed, such as tetrahedral *hp* finite elements [1], the spectral volume method [2], or the spectral method for the shallow water equations [3]. High order or *spectral* methods are usually referred to as methods of cubic convergence and above [4], distinguishing from low-order methods, of the first and second order of convergence. Remaining in the field of CFD, the finite volume (FV), or control volume (CV), method is one of the most widely used techniques. Among the attractive features of the CV approach are quoted the excellent numerical conservation properties, the intuitive physical formulation and the relatively easy implementation. Following these considerations, the proposed algorithm [5] incorporates the desirable physics-conserving properties of the CV method with the capability of arbitrary straightforward high order accuracy distinctive of the *discontinuous* finite-element (FE) approach, resulting in the hybrid discontinuous control-volume/finite-element method (DCVFEM). The present work provides the extension of the spectral DCVFEM to the solution of the Euler equations in the one-dimensional case. The method is presented in its main features, and numerical experiments are conducted to verify the expected improved stability and accuracy in shock-capturing. The solution is compared to the well established discontinuous Galerkin FE method (DGFEM) and the FV method.

## References

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