

# SHAKEDOWN ANALYSIS OF 3D FRAMES WITH AN EFFECTIVE TREATMENT OF THE LOAD COMBINATIONS

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In the last two decades, the interest in direct methods to be applied to limit and shakedown analysis has rapidly increased, principally due to the availability of highly efficient new optimization algorithms. The Interior Point revolution has completely changed the way of solving convex programming and, today, these algorithms are efficiently employed for very large non-linear problems. Nowadays, the best performance of the Interior Point Method (IPM) is obtained when the problem is formulated as conic programming and the solution is obtained using primal-dual formulations [1, 2]. A great number of yield constraints can be described as second-order cones allowing the proposal of efficient interior point algorithms for limit and shakedown analysis.

Alternatively, a specialized direct method, named pseudo elastoplastic analysis, can be used to evaluate the limit and shakedown safety factors. This approach, see [3, 4], is based on a strain-driven strategy of analysis hinged on closest point projection return mapping schemes and Riks arc-length solution techniques. This method can be seen as the application of the proximal point algorithm to the static shakedown or limit analysis theorem and the solution of the resulting problem is performed by means of the dual decomposition strategy [4].

Independently from how the shakedown problem is solved, its practical application to the analysis of 3D frames requires a fine tuning of two important aspects: i) an accurate and simple definition of the section yield function; ii) the limitation of the number of load combination to be considered.

The yield function of 3D frame is usually evaluated only considering flexural failures. In spite of this simplifying assumption, computing accurate yield surfaces with combined axial force and bending moments is not an easy task and has received increasing attention in the literature [5, 2]. A piecewise linearization often requires a large number of polyhedral facets to obtain a sufficiently accurate approximation, which can have an important effect

on the quality of the estimated bounds [5] but also on the efficiency of the algorithm. Since the yield criterion has to be verified for a large number of points throughout the whole structure, a compromise between accuracy and computational efficiency is required in the case of large-scale problems. This aspect becomes also more important for shakedown analysis where the number of constraints depends on the number of basic loads exponentially [3, 4].

Recently a strategy for treating 3D frames describing them through nonlinear yield surfaces was suggested in [2] in the context of limit analysis, where the real yield surface is approximated by using ellipsoids. In this way the arising optimization problem becomes a SOCP problem which can be efficiently solved also with a commercial code such as MOSEK. A similar approach is also adopted in the present work allowing to accurately describe the material elastic domain and to use only few analytical yield functions. However due to the several load conditions to be considered, the number of constraints required for the shakedown analysis of 3D frame can be very large. In the more simple case of load domain defined by means of a sum of  $n$  basic loads varying between a minimum and a maximum value we have to verify the plastic admissibility for each of the resulting  $2^n$  possible vertexes of the convex polytope defined by the elastic stresses associated to each load, that is for each vertex of the so called elastic envelope polytope. The number of vertexes of the elastic envelope heavily affects the computational performance of the analysis whatever method is employed, a standard direct formulation, such as the interior point algorithm used in [1] or, also with a minor impact, the pseudo elastic-plastic formulation.

In the paper we propose an efficient and effective strategy to select for each finite element or Gauss point, i.e. where the plastic admissibility condition has to be tested, only a subset of vertexes without affecting the accuracy of the shakedown analysis.

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