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SHEAR DEFORMABLE HYBRID FINITE-ELEMENT FORMULATION FOR BUCKLING ANALYSIS OF THIN-WALLED MEMBERS

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Thin-walled structural elements are used in industrial and residential structures such as bridges, ship hulls, aircrafts and aerospace buildings. Because of their slenderness, which leads to susceptibility to buckling, their response should be predicted accurately. Closed-form solution procedures for the buckling analysis of thin-walled members are limited to certain boundary conditions and type of loading. Consequently, numerical methods have been developed to be able to address general cases.

The effect of shear deformation can gain importance in the buckling behaviour of beams especially with built-up or composite sections or alternatively when materials with relatively low shear modulus is used such as FRP. In displacement based formulations the kinematic assumptions of Vlasov [1]should be modified to include the shear deformation effects. On the other hand, shear deformations can be included by using the strain energy of the shear stresses in a complementary-energy based formulation [2] without modifying the kinematic assumptions. However, in a complementary energy based formulation, the inter-element force equilibrium conditions have to be satisfied a-priori, which complicates the assemblage procedure [3]. Mixed-hybrid finite element formulations can be used to overcome the shortcomings of both displacement based and complementary energy based formulations.

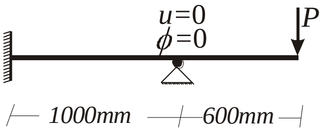
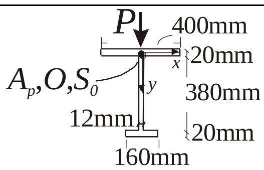
In this study, by introducing the equilibrium and force boundary conditions as auxiliary conditions to the complementary energy functional, the Hellinger-Reissner functional is obtained. The complementary energy that is adopted in this study is the one introduced by Koiter [4], in which the Jaumann stress tensor for finite elasticity solutions is adopted.

Examples are presented herein to show the applicability of proposed formulations to practical problems and to discuss the importance of shear deformation effect. In these examples, two elements are developed: BEH (Buckling Element based on Hybrid formulation), in which shear deformation is omitted and BEHS (Buckling Element based on Hybrid formulation including Shear deformation), where the effect of shear deformation is considered. In addition, shell element model developed in ABAQUS commercial software is provided for comparison purposes.

The material properties are initially E=200GPa for the elasticity modulus and G=77GPa for the shear modulus and thus E/G ratio is 2.6 based on which the buckling load was determined by the eight element BEH solution as 9343kN. The buckling analysis based on eight element BEHS predicts 8499kN. Thus, the difference between the BEH and BEHS solutions is around

10% due to the effect of shear deformation. In [3], by using an ABAQUS shell element model, the first mode of buckling was verified to be an overall lateral-torsional buckling mode and thus, despite a short span the beam local buckling modes did not dominate the design.

Table 8 Buckling load predictions for a propped cantilever with an overhang

Boundary Conditions and Loading	Cross-section	ABAQUS (kN)	BEH (kN)	BEHS (kN)
		7,180	9,343 (%130)	8,499 (%118)

On the other hand, when the *E*/*G* ratio is increased to 26 by reducing the shear modulus ten times, which is a typical value for FRP pultruded beams, the buckling load predictions based on the BEHS reduce from 8499kN to 4551kN (a reduction of some 47%). It should be noted that when the shear deformation effects are excluded, BEH predicts the buckling load as 8135kN which is 44% higher than 4551kN and thus a significant overestimation occurs for a beam consisting of a material with low shear modulus.

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