

FINITE DEFORMATION BEAMS FOR PROBLEMS OF INTERACTION

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In many nonlinear coupled problems, solids, often slender in geometry, are in interaction with complex three-dimensional fields. Such is the case for electrostatically actuated micro- and nano-scale beams which serve as the core part in many micro and nanoelectromechanical systems (MEMS/NEMS) based applications. Interaction loads take the form of normal tractions on the interface that arise from electrostatic “pressure” distribution. Another interesting application is the analysis of beam-like structures fabricated from ionic polyelectrolyte hydrogels. In these materials, which are hydrophilic polymer networks responsive to electrical and chemical stimuli, the interaction loads (chemoelectromechanical coupling) take the form of an applied swelling stress distribution engendered by the electrically controlled ion migration within the gel. In the following, the non-solid part of these two types of coupled problems is generally labeled chemo-electrostatic for simplicity, although in the MEMS devices it is purely electrostatic.

In these coupled problems, chemo-electrostatic forces load the solid, and structural deformation in turn alters the geometry of the chemo-electrostatic domain. Nonlinear effects become more pronounced with scale. The common, yet computationally intensive, approach for solving these problems is to consider three-dimensional elastic and chemo-electrostatic continua. The computational cost can be reduced considerably by the use of structural models for the solid.

At the onset, a geometrically exact beam model is considered. Geometrically exact beam theory has, as its only kinematic restriction, the classical assumption of rigid cross sections, routinely accompanied in practice by modified constitutive equations akin to those found in low-order linear beam theories. We express beam counterparts of applied loads in terms of the underlying three-dimensional data, accounting for the various types of continuum chemo-electrostatic forces. This model is suitable for the normal traction loading typical of MEMS devices. However, the applied stress distribution in the hydrogel structures gives

rise to large cross-sectional deformation, which is not represented by the geometrically exact beam.

We extend the geometrically exact beam model to allow distortion of the cross section, by introducing an appropriate set of cross-sectional basis functions. The inclusion of cross-sectional deformation allows straightforward application of three-dimensional constitutive laws in the beam formulation. Numerical comparisons show the ability of this beam model to reproduce finite elasticity results with good efficiency.

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