A Multiphase MPS Formulation for Bubble Flow

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Introduction

Numerical simulation of multiphase flows is one of the most challenging issues in CFD because of their complicated physics. The motion of interface plays an important role in multiphase flows because of the discontinuity of fluid properties and possible topological changes such as break up and coalescence. Particle methods appear to be appropriate candidates for the simulation of multiphase flow because interfacial deformation can be easily tracked by the Lagrangian motion of particles.

In this paper, the Moving Particle Semi-implicit method (MPS) is extended to model the multiphase flow. The density jump between different phases relates closely with the pressure field, so the derivation of Pressure Poisson Equation (PPE) in MPS is revisited for multiphase flow. A new surface tension model based on the surface tension formulations in Front Tracking Method and Continuous Surface Force model is proposed and it can effectively prevent particles from penetrating into the other phase.

Method and Results

For multiphase flow simulation, ρ is no longer homogenous constant but a variable, and the Pressure Poisson Equation (PPE) is found to be

$$\nabla \cdot \left(\frac{1}{\rho} \nabla P\right) = -\frac{1}{\Delta t^2} \left(\frac{n^* - n_0}{n_0}\right) \tag{1}$$

The left hand side of the above Poisson equation can be discretized as follows,

$$\nabla \cdot \left(\frac{1}{\rho} \nabla P\right) = \frac{2D}{n_0 \lambda} \sum_{j \neq i} \frac{P_j - P_i}{(\rho_j + \rho_i) / 2} w(|\mathbf{r}_j - \mathbf{r}_i|)$$
(2)

where *i* and *j* are labels for the reference and neighbor particle respectively. The PPE matrix coefficients in liquid phase are small because the liquid density is large while those coefficients in gas phase are relatively large because the gas density is small. So the solved pressure field varies heavily in liquid phase but varies slightly in gas phase. The different variation of pressure field in different phases produces buoyancy for the lighter phase so it is not necessary to add extra buoyancy term[1]. Meanwhile, the averaged density of particle *i* and *j* in Eq. (2) can help smooth the matrix coefficient across the interface so that the solved pressure field is also smooth.

A Rayleigh-Taylor problem ($\rho_{\text{green}}=100\text{kg/m}^3$, $\rho_{\text{pink}}=1000\text{kg/m}^3$) is simulated to verify our model. In the simulation, the mixing of the two fluids is induced by random fluctuation in the MPS method and the initial interface fluctuations are not used.



Fig.1 Simulation of Rayleigh-Taylor instability at 0.2s

The original Continuous Surface Force (CSF) model in MPS sometimes cannot prevent particle of one phase from penetrating into the other phase. In our work, we combine the surface model used in Front Tracking Method[2] and the original CSF model together to proposed a new surface model, which can bind the interfacial particles tightly. Such a surface tension model can produce stable and sharp interface as shown by droplet deformation ($\rho_l = \rho_d = 1000 \text{kg/m}^3$, $\sigma = 0.07 \text{N/m}$, $D_{\text{droplet}} = 5 \text{mm}$) in Fig. 2. It is possible to simulate bubble coalescence ($\rho_l = 1000 \text{kg/m}^3$, $\rho_g = 1 \text{kg/m}^3$, $\sigma = 0.07 \text{N/m}$, $D_{\text{bubble}} = 5 \text{mm}$) by the extended MPS method as shown in Fig.3.



Fig.2 Droplet deformation



Fig.3 Bubble Coalescence

Conclusion

In this work, the MPS method is extended to multiphase flows by introducing new formulation of pressure term and surface tension model. The method can handle bubble flows characterized by high-density ratio and strong surface tension.

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