11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

DDM APPLIED TO SUBSURFACE FLOW AND TRANSPORT

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Key Words: *Computing Methods, Domain Decomposition Methods, Derived Vector Space*

In this work various mathematical and numerical techniques were developed to apply domain decomposition methods, DDM. Applying this method and the different numerical methods, to the flow and transport in porous media, makes possible to obtain efficient parallelization of the governing equations in reservoirs with dominant advection.
The domain decomposition method has been investigated recently by several authors for bi-dimensional and tri-dimensional elliptic and parabolic problems. This method is attractive because it permits parallel processing of finer meshes to approach the domain at transport problems.
On the base of the consideration that DDM are the most effective manner of applying parallel processing to the solution of partial differential equations, this work presents a non-overlapping DDM: the 'derived-vector space', introduced in recent years [1-4], and that supplies a framework that is very suitable for the unified treatment, by non-overlapping DDMs, of symmetric, non-symmetric and indefinite systems.
The derived-vector space (DVS) is a finite-dimensional linear space, which is a realization of a product-space. It constitutes a Hilbert-space with respect to an inner product that is defined independently of the partial differential equations to be treated. Thus, it is a setting that exists by itself independently of the problems considered and can be used as a framework in which any problem can be defined and in its realm can also be treated by any method. Therefore, the derived-vector space is a suitable setting for developing a general unifying theory of non-overlapping DDMs. In it, non-symmetric and indefinite systems can be treated. Primal approaches, i.e., direct methods that do not use Lagrange multipliers as well as dual approaches, i.e., indirect methods that use Lagrange multipliers, can be applied to them. In this manner a theory has been developed, which is equally applicable to symmetric definite matrices, non-symmetric and indefinite matrices, and in which primal and dual approaches are incorporated.
Among the algorithms of non-overlapping domain decomposition methods that are used at present, the balancing domain decomposition, BDD, and its variants, is the prototype of primal approaches; while the finite-element tearing and interconnecting, FETI, and its variants, is the prototype of dual approaches. In particular, the most prestigious primal algorithms that exist today are the BDD algorithms with constraints (BDDC) introduced by Dohrmann [5] and further developed jointly with Mandel and others [6]. Correspondingly, the most prestigious dual algorithms are the dual-primal FETI (FETI-DP) algorithms introduced by Farhat [7] and have received much attention by many authors.
The principal feature of the DVS-framework is that, when a method is applied in its area, the

original problem is transformed into an equivalent one defined in the derived-vector space, which as mentioned before is a product-space, and thereafter all the work is carried out in it. Nothing like that is done in other formulations. In BDDC formulations, for example, the original space of continuous functions is never completely abandoned; indeed, that application frequently goes back and forth from the degrees of freedom associated with the original space of continuous functions to the degrees of freedom associated with the substructures, which in such formulations play the role of a product-space.

This approach yields simple unified matrix-expressions, in terms of a generalized Schur-complement matrix. In this work it is considered some simple iterative sub-structuring methods that rely on a partition of non-overlapping sub-domain. At the global domain an internal boundary at local domains are defined. Various mathematical and numerical methods were developed to apply the multipliers-free DDMs. The Schur complement matrix, relative to the unknowns on the internal boundary is obtained. This matrix can be found by sub-assembling local contributions. In particular solving the Schur system for the unknowns at the internal boundary, the internal components can be found. Applying this method, to the flow and transport in porous media, permit to obtain efficient parallelization of the governing equations in reservoirs with dominant advection.

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