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# Singular stress analysis of sharp three-dimensional interfacial corner of jointed dissimilar materials using H-integral

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Three-dimensional interfacial corner is one of the main causes of fracture in electronic packages, MEMS and so on. In our previous study, we analyzed quasi-three-dimensional interfacial corner of jointed anisotropic piezoelectric materials using the Stroh formalism and H-integral. In this case, the corner front is smooth and the stress in the vicinity of the corner point can be assumed plain strain condition. However, the fracture in the jointed structures occurs from a sharp three-dimensional corner that cannot be assumed to be two-dimensional problem even in the vicinity of the corner.

Pageau and Bigger [1] proposed a numerical method to obtain the stress singularity orders of a three-dimensional interfacial corner between jointed dissimilar materials. We used their method to obtain the stress singularity orders of three-dimensional jointed corners. However, the asymptotic solution around the corner is expressed by the scalar parameters, which express the magnitude of the stress, and the stress singularity orders as

$$\sigma_{ij}^{k} = \sum_{m=1}^{N} C_m r^{\lambda_m - 1} f_{ij}^{mk}(\theta, \phi) \qquad (i, j = 1,2,3),$$

where $\sigma_{ij}^{k}$ is stress in $k$-th material, $C_m$ is scalar parameters, $r$ is the distance from the corner, $1\text{-}\lambda_m$ is the stress singular orders and $f_{ij}^{mk}$ is the eigen function. $\theta$ and $\phi$ are the polar coordinates around a sharp corner between jointed dissimilar materials as shown in Fig. 1.

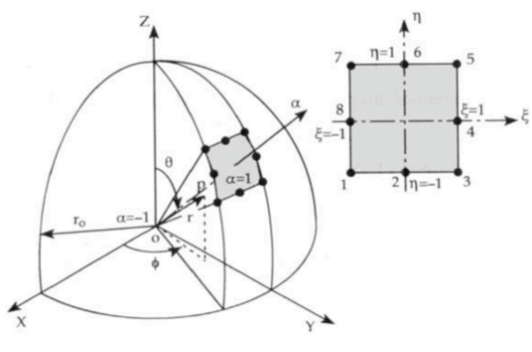


Figure 1 Polar coordinates around a sharp corner between jointed dissimilar materials.

We propose a new method to obtain the scalar parameters of three-dimensional jointed corners using the *H*-integral method. The *H*-integral method, which is derived from Betti's reciprocal principle, is useful for analyzing the stress intensity factors (SIFs) of cracks and corners. We expanded this method to obtain the scalar parameters of a sharp three-dimensional interfacial corner between anisotropic multi-materials. This method has high generality that can deal a jointed corner with various numbers of dissimilar materials and several boundary conditions on the corner surfaces. We demonstrate the accuracy of the singularity stress field expressed by the asymptotic solutions with the obtained stress singularity orders and scalar parameters.

**REFERECES**

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