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# A DISCONTINUOUS GALERKIN METHOD FOR NON-NEWTONIAN IMPLICIT CONSTITUTIVE MODELS OF NONEQUILIBRIUM GASES

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The discontinuous Galerkin (DG) method [1-3] has been popular as a computational method for solving the hyperbolic conservation laws of gas dynamics. The DG method has recently found its way into the main stream of CFD as an alternative approach for CFD based on the traditional finite volume framework. The DG method combines key features of the finite element and finite volume methods, and has been successfully applied to a variety of problems, such as gas dynamics, acoustics, and magneto-hydrodynamics. It may be divided into modal or nodal, depending on the basis function used in the scheme.

There are growing interests in applying the DG method to new branch of engineering and science. In the present work, we consider a novel application to the field of rarefied and microscale gases. These new gas flows may be found in three distinctive problems: 1) the hypersonic rarefied regime of trans-atmospheric vehicles; 2) the microscale low-speed (creeping) regime of micro- and nano-devices; and 3) the high-speed free-molecular regime of mechanical devices operating near the vacuum condition. From past experiences, it was accepted that conventional models based on classical physics, such as the NSF equations, have serious limitations in capturing the correct flow physics of high thermal nonequilibrium. In order to overcome this shortcoming, a model called nonlinear coupled constitutive relation (NCCR) in a compact implicit algebraic form [4] was derived by starting from the generalized hydrodynamic equations pioneered by Eu. An important result obtained in previous studies was that constitutive relations between stresses (heat flux) and the strain rate (the temperature gradient) are generally nonlinear and coupled in states removed from thermal equilibrium.

In this work, we present an explicit modal DG scheme for solving multi-dimensional hyperbolic conservation laws in conjunction with the NCCR models. The main emphasis is placed on how to treat the complex non-Newtonian type constitutive relations arising from the high degree of thermal nonequilibrium in multi-dimensional gas flow situation within the discontinuous Galerkin framework. For the verification and validation study, we apply the scheme to the problems of the very stiff 1D shock wave structure for all Mach numbers and the 2D hypersonic rarefied and low-speed microscale gas flows past a circular cylinder. In particular, we present the numerical solutions of the compressible low Mach number gas flow with high Knudsen number, one of most studied problem in microscale gas dynamics and yet remaining as extremely challenging from numerical viewpoint since the conventional NSF-

FVM scheme and the DSMC suffer very poor accuracy and extraordinarily slow convergence.

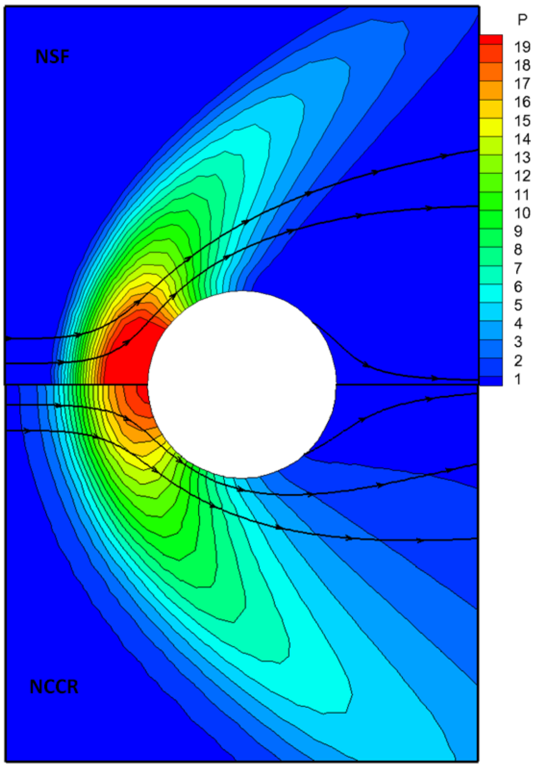


Figure 1: Normalized pressure fields and contours in the hypersonic case, $M$ = 5.48 and Kn = 0.5.

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