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RELIABILITY-BASED OPTIMIZATION APPLYING POLYNOMIAL CHAOS EXPANSION

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Reliability-based Design Optimization is achieving more and more agreement in the industrial design community. In fact, most of the industrial processes are permeated by uncertainties: the manufactured product is generally different, from a geometric point of view, from the product design because of the dimensional tolerances, and, more frequently, the working point is not fixed, but is characterized by some fluctuations in the operating variables.
This uncertainty is commonly transferred to the performance of the system, which cannot be determined with an exact and single value, but which is better described by a statistical distribution of results.
In this environment, a frequent industrial requirement is the satisfaction of **constraints or limits**, which should be achieved for a specified percentage of the performance distribution, or for which the percentage of solutions not satisfying the limits (**failure probability**) must be minimized as much as possible, to improve the reliability and quality of the product [1].

In literature, there are basically two types of approaches for this kind of problems.
One approach is the **Robust Design Optimization** [1,2], commonly followed by many industries employing software like **modeFRONTIER**, which basically consists in evaluating, for each candidate design proposed by the optimization algorithm, the stochastic distribution of its performances, and in defining objectives based on mean and standard deviations of the same (for instance, maximize mean performances and minimize their standard deviations, in order to optimize the stability at the fluctuations). The strategy is particularly efficient, also because it takes now advantage of Polynomial Chaos expansion [3], an efficient methodology which exploits proper ortho-normal Polynomials (figure 1, left) to estimate analytically therefore with high accuracy the main moments of the performance distribution, i.e. mean and standard deviation, through a reduced number of sampling evaluations.
The application limit of this methodology for the reliability-based problems as described above is indeed the fact that mean and standard deviation may be not enough to compute the complete stochastic distribution of the performance in particular when it is not Normal, therefore it is not possible to find the exact failure probability for any prescribed limit.

The other approach followed in literature is the Reliability analysis which implements methodologies like **FORM or SORM** [4], which evaluate the failure probability of any candidate design on the basis of its uncertainties distribution and of the given limits to be respected. One limit of this methodology can be represented by the **high number of evaluations** that may be required by the algorithm to compute the failure probability with accuracy, which makes often practically unfeasible its application to optimization problems of industrial relevance.

For these reasons, we propose in this paper a **new methodology** to deal efficiently with a reliability-based industrial optimization problem, which conjugates accuracy and small number of needed evaluations.
The methodology is derived from the first approach described above, i.e. Robust Design Optimization applying **Polynomial Chaos**, which means that for each candidate design proposed by the optimization algorithm, Polynomial Chaos is applied to a small sampling set. The polynomial coefficients are at this point used to evaluate the complete **cumulative distribution function of the performances** of the design, from which it is possible to retrieve accurately the **failure probability** for the prescribed objectives/constraints.

The presentation will include the theoretical details of the methodology, and will propose a validation and an application case. A first **mathematical problem** (deflection of a cantilever beam) will be first proposed in order to describe the problem and to compare the results obtained by the traditional (FORM) and by the new approach, highlighting the accurate convergence of the results and the advantages in terms of number of simulations required by the new approach (fig.1 right). The new approach is then applied to a reliability industrial problem, which consists in the design optimization of an **Heat Exchanger** by experimental correlations, under operational uncertainties and geometrical tolerances.

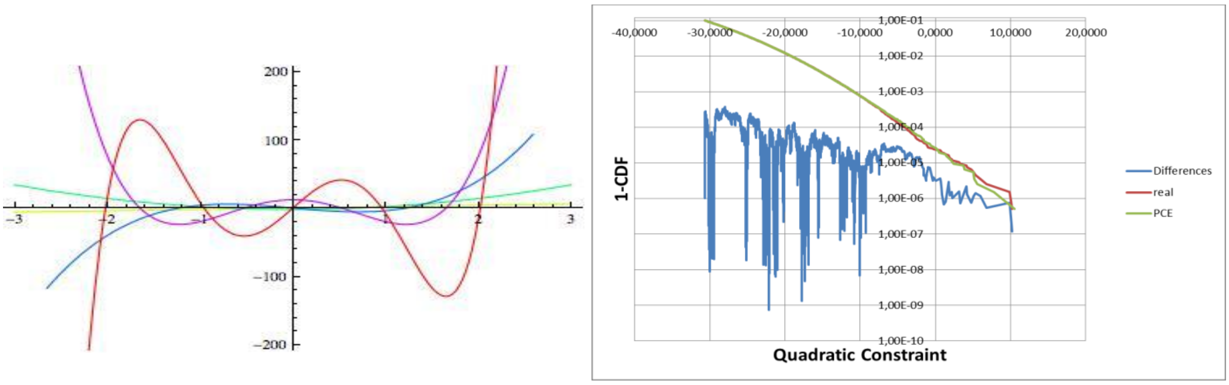

Figure 1. Polynomial Chaos (left); CDF constraint predicted by Polynomial Chaos vs exact solution (right)

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