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IMPROVED REDUCED ORDER MODELS FOR THE COMPUTATION OF HOPF BIFURCATIONS IN FLUID MECHANICS

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This work deals with the computation of Hopf bifurcations in 2D Navier-Stokes equations. These bifurcation points correspond to an instability of the fluid flow which is characterized by the transition to a stationary state towards an instationary one. These bifurcation points are determined by using a Hybrid method[1]. This latter associates an indicator curve and a Newton method[2]. The indicator provides initial values for the Newton method. As the calculus of this indicator is time consuming, we propose, in Reference [3] an algorithm to decrease these computational times. This algorithm alternates reduced order and full size steps resolution which are all carried out by using a pertubation method. Hence, the computed vectors on the full size problem are used to define the reduced order model. As, the low-dimensional model has a finite validity range, we propose a simple criterion allowing to know when the basis has to be updated. This latter phase is realized by doing a new full step which permits to build a new basis and so compute a supplementary part of the indicator curve. The aim of this work is to improve the validity range of the reduced solutions and then limit the required number of full size matrix triangulations. This is carried out by adding a linear correction of the reduced solution. This correction is done by a simple iterative algorithm which used a preconditionner matrix. This latter is the triangulated matrix of the full size computation. So, each full size computation gives firstly the reduced basis and secondly the preconditionning matrix. This method is applied to a classical example in fluid mechanics, the 2D lid-driven cavity. In Fig. (1), the evolution of the bifurcation indicator, Ref. [1], is plotted versus the angular frequency. In Fig. (1-a), the solution obtained with the reduced order model defined in Ref. [3] is plotted (the green curve). In Fig. (1-b), the reduced solution associated with a correction phase is plotted (the green curve). In these two figures, the red curve is the reference curve obtained without reduction technique[1]. In Table 1, the number of full-size steps, to get the indicator curve of the figure (1) is given for each method. Results in this table show that the proposed reduced method (with the correction phase) is the method requiring the smallest number of matrix triangulation

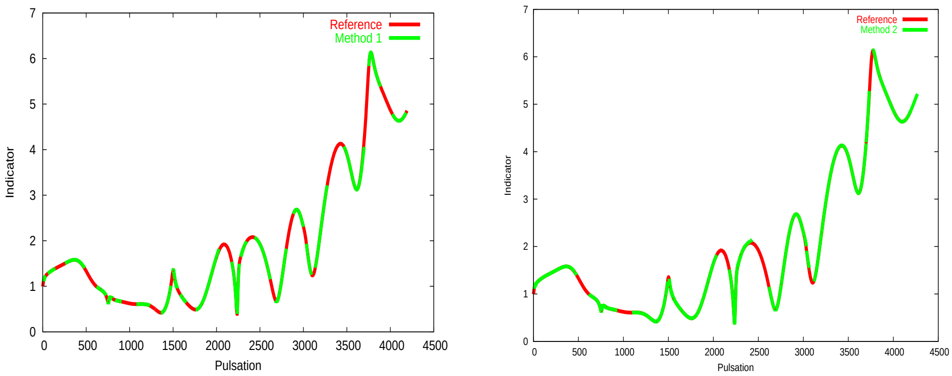
	Number of full steps
Reference [1]	66
Heyman *et al* [3]	22
Proposed Method	10

Table 1: Number of full steps to compute the indicator curve of the Fig. (1).

Re_{c1}	Str_1	Re_{c2}	Str_2	Re_{c3}	Str_3	Re_{c4}	Str_4	Re_{c5}	Str_5	Re_{c6}	Str_6
7890	0.44	8829	0.52	8921	0.61	9513	0.7	11286	0.33	11417	0.78

Table 2: Lid driven cavity. Hopf bifurcation points (Reynolds and Strouhal numbers) determined with the proposed technique.

and then the smallest computing time. Finally, all the minima of indicator (determined with the curve in Fig. 1) are introduced into a Newton method[2] and permit to compute the bifurcation points summarized in Table 2.



(a) Method from [3]

(b) Proposed Method

Figure 1: Lid-driven cavity. Indicator versus angular frequency. Red and green color curves stand for respectively full and reduced order models.

REFERENCES

[1] A. Brezillon, G. Girault and J.M. Cadou, A numerical algorithm coupling a bifurcating indicator and a direct method for the computation of Hopf bifurcation points in fluid mechanics, *Computers & Fluids*, Vol. **39**, 1226-1240, 2010.

[2] C. P. Jackson, A finite-element study of the onset of vortex shedding in flow past variously shaped body, *J. Fluid Mech.*, Vol. **182**, 23-45, 1987.

[3] J. Heyman, G. Girault, Y. Guevel, C. Allery, A. Hamdouni and J.M. Cadou, Computation of Hopf bifurcations coupling reduced order models and the asymptotic numerical method *Computers & Fluids*, Vol. **76**, 73-85, 2013.