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FULL C^1 CONTINUITY MULTISCALE SECOND-ORDER COMPUTATIONAL HOMOGENIZATION APPROACH

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In recent years, a special attention has been directed to investigate relations between the macroscopic properties of engineering materials and their microstructure. It is well known that the classical continuum mechanics cannot consider structural effects in the material at the microlevel and therefore, multiscale techniques for modeling on multiple levels using homogenization procedures have been developed. Several homogenization methods have been proposed, where the second-order computational homogenization scheme has been proven as most versatile tool [1-3], enabling consideration of the size effect. The multiscale algorithm comprising second-order homogenization procedure requires C^1 continuity at the macrolevel and employs nonlocal continuum theory. The discretization of representative volume element (RVE) at the microlevel is usually performed preserving the standard C^0 continuity and keeping classical boundary value problem with standard material models. However, due to the C^1 - C^0 transition, some inconveniences arise. Firstly, the microlevel second-order gradient cannot be related to the macrolevel as volume average, since such relation requires higher-order boundary conditions. Therefore, to prescribe full macrolevel second-order gradient, an alternative relation is considered, which leads to derivation of microfluctuation boundary integrals involving RVE boundary nodal displacements [1]. As the second, the minor discrepancy exists in Hill-Mandel energy condition, since higher order variables are not included into microlevel computation, which requires a modified definition of the second-order stress as first moment of the Cauchy stress tensor. In this way, such approach does not satisfy volume average equivalence between the micro and macro second-order stress tensors.

To overcome these shortcomings, the present contribution deals with a multiscale second-order computational homogenization algorithm keeping the C^1 continuity at both the macrolevel and the microlevel under assumptions of small strains and linear elastic material behavior. Accordingly, the computational models at both levels are discretized by the C^1 continuity three-node plane strain triangular finite element, developed in [4] and reformulated for multiscale analysis in the authors' previous papers [5, 6]. To accelerate the solution process of the multi-scale problem, the reduced numerical integration scheme is used to evaluate the element matrices. Herein, a new scale transition methodology is derived in which the volume average of every macrolevel variable prescribed at the microlevel is explicitly satisfied without using boundary integrals to enforce equality of micro- and macro- variables. Furthermore, employing the Hill-Mandel energy condition the macrolevel stress tensor is extracted as volume average of microlevel stress tensor, without auxiliary definition of higher-order stress. The macroscopic consistent constitutive matrix is computed from the RVE global stiffness matrix using the standard procedures. A special attention is directed to the application of the gradient displacement- and generalized gradient periodic boundary conditions on the RVE. The new proposed full C^1 continuity multiscale second-order homogenization scheme is presented in Fig. 1. The algorithms derived are implemented into FE software ABAQUS via user subroutines. Finally, in order to test the performance of the proposed multiscale homogenization approach, several standard numerical examples are considered.

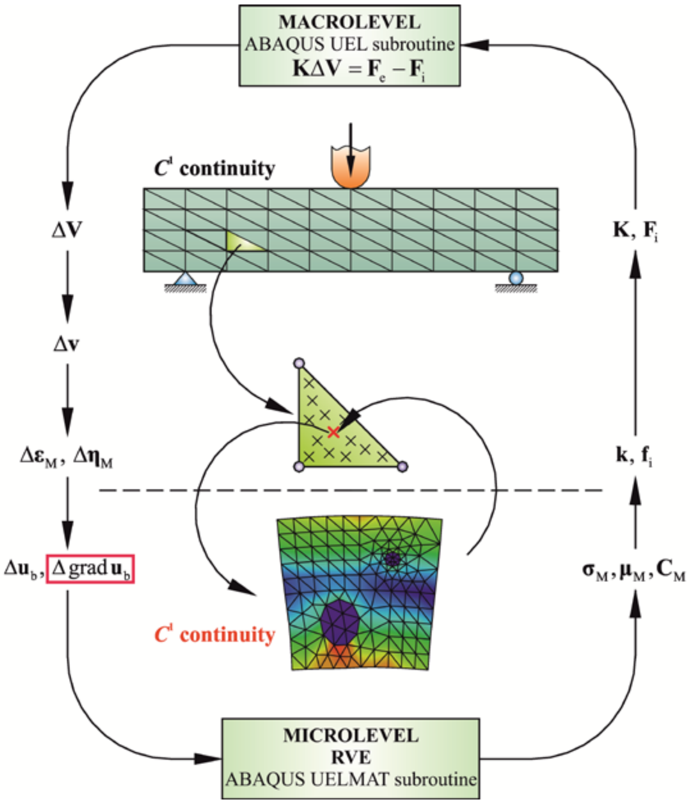

Figure 1. Scheme of the full C^1 continuity multiscale second-order homogenization approach

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