DISCRETE RELIABILITY FOR CROUZEIX–RAVIART FEMS

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The proof of optimal convergence rates of adaptive finite element methods relies on Stevenson’s concept of discrete reliability [2]. This presentation presents a proof of the general discrete reliability for the nonconforming Crouzeix-Raviart finite element method on multiply connected domains in any space dimension.

The discrete reliability states that the difference of the discrete solutions on two arbitrary levels \( u_\ell \) and \( u_{\ell+m} \) with respect to triangulations \( \mathcal{T}_\ell \) and \( \mathcal{T}_{\ell+m} \) is bounded by the contributions of the residual-based error estimator on the refined simplices \( \mathcal{T}_\ell \setminus \mathcal{T}_{\ell+m} \) only. After some natural split of the error, the additional difficulty for the nonconforming FEMs is to bound

\[
\min_{v_{\ell+m} \in \text{CR}(\mathcal{T}_{\ell+m})} \| \nabla_{\text{NC}} (u_\ell - v_{\ell+m}) \|_{L^2(\Omega)}^2
\]

by weighted jumps of \( u_\ell \) on refined hyper-surfaces only. The underlying paper [1] of this presentation provides a rigorous proof of this discrete distance control for multiply connected domains \( \Omega \subseteq \mathbb{R}^n \) in any space dimension \( n \geq 2 \). The main tool is the definition of a novel transfer operator which is based on an intermediate triangulation.

This enables the discrete reliability for a couple of model problems like the Poisson problem, eigenvalue problems, Stokes equations and linear elasticity and thereby shows optimal convergence of the AFEM for those problems in the general case.

REFERENCES
