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DISCRETE RELIABILITY FOR CROUZEIX–RAVIART FEMS

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The proof of optimal convergence rates of adaptive finite element methods relies on Stevenson's concept of discrete reliability [2]. This presentation presents a proof of the general discrete reliability for the nonconforming Crouzeix-Raviart finite element method on multiply connected domains in *any* space dimension.

The discrete reliability states that the difference of the discrete solutions on two arbitrary levels u_ℓ and $u_{\ell+m}$ with respect to triangulations $\mathcal{T}_\ell$ and $\mathcal{T}_{\ell+m}$ is bounded by the contributions of the residual-based error estimator on the refined simplices $\mathcal{T}_\ell \setminus \mathcal{T}_{\ell+m}$ only. After some natural split of the error, the additional difficulty for the nonconforming FEMs is to bound

$$\min_{v_{\ell+m} \in \mathrm{CR}(\mathcal{T}_{\ell+m})} \|\nabla_{\mathrm{NC}}(u_\ell - v_{\ell+m})\|^2_{L^2(\Omega)}$$

by weighted jumps of u_ℓ on refined hyper-surfaces only. The underlying paper [1] of this presentation provides a rigorous proof of this discrete distance control for multiply connected domains $\Omega \subseteq \mathbb{R}^n$ in any space dimension $n \geq 2$. The main tool is the definition of a novel transfer operator which is based on an intermediate triangulation.

This enables the discrete reliability for a couple of model problems like the Poisson problem, eigenvalue problems, Stokes equations and linear elasticity and thereby shows optimal convergence of the AFEM for those problems in the general case.

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