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IMPROVEMENT OF BALANCING DOMAIN DECOMPOSITION METHOD FOR PROBLEM WITH MULTI-POINT CONSTRAINTS

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One of the authors proposed a method to incorporate multi-point constraints (MPCs) into the balancing domain decomposition (BDD) method [1]. A purpose of this study is to improve convergence property of the method. The method is implemented in a parallel finite element structural analysis code, ADVENTURE_Solid. It is used in the project of K Computer (HPCI Program Fields 3 and 4) for the large-scale seismic response analysis of steel frames. Outline of the method is as follows. A set of MPCs that are enforced on the nodal displacement vector $\mathbf{u}_\mathrm{B}$ for interface degrees of freedom of the BDD method is represented as follows:

$$\mathbf{B}^\mathrm{T}\mathbf{u}_\mathrm{B} = -\mathbf{r} \tag{1}$$

where $\mathbf{B}$ is the constraint matrix and $\mathbf{r}$ is a constant vector. Note that the internal nodes in subdomains on which one or more MPCs are enforced are converted into interface nodes. The constraints on the interface problem are imposed using the Lagrange multiplier method, and the interface problem is solved by the conjugate projected gradient method. The preconditioner for the BDD method is represented as follows:

$$\mathbf{M}_\mathrm{BDD}^{-1} = \left(\mathbf{I} - \mathbf{R}\mathbf{K}_\mathrm{C}^{-1}\mathbf{R}^\mathrm{T}\mathbf{S}\right)\mathbf{M}_\mathrm{NN}^{-1}\left(\mathbf{I} - \mathbf{S}\mathbf{R}\mathbf{K}_\mathrm{C}^{-1}\mathbf{R}^\mathrm{T}\mathbf{S}\right) + \mathbf{R}\mathbf{K}_\mathrm{C}^{-1}\mathbf{R}^\mathrm{T} \tag{2}$$

where $\mathbf{M}_\mathrm{NN}^{-1}$ is the preconditioning matrix of the Neumann–Neumann preconditioner, and $\mathbf{K}_\mathrm{C}$ is the stiffness matrix of the coarse grid problem. $\mathbf{K}_\mathrm{C}$ is calculated as follows:

$$\mathbf{K}_\mathrm{C} = \mathbf{R}^\mathrm{T}\mathbf{S}\mathbf{R} \tag{3}$$

where $\mathbf{S}$ is the Schur complement, $\mathbf{R}$ and $\mathbf{R}^\mathrm{T}$ are the prolongation and restriction matrices, respectively. The matrix $\mathbf{R}$ for a subdomain k is reresented as follows:

$$\mathbf{R}^{(k)} = \mathbf{R}_\mathrm{B}^{(k)}\mathbf{D}^{(k)}\mathbf{Z}^{(k)} \tag{4}$$

where $\mathbf{R}_\mathrm{B}^{(k)}$ is a Boolean matrix that maps the global interface DOFs to the local interface DOFs, $\mathbf{D}^{(k)}$ is a wieght matrix, and $\mathbf{Z}^{(k)}$ represents a local coarse space. In the structural analysis, a part of the matrix $\mathbf{Z}^{(k)}$ that corresponds to a node P is represented as follows:

$$\mathbf{Z}_P^{(k)} = \begin{bmatrix} 1 & 0 & 0 & 0 & x_3 & -x_2 \\ 0 & 1 & 0 & -x_3 & 0 & x_1 \\ 0 & 0 & 1 & x_2 & -x_1 & 0 \end{bmatrix} \tag{5}$$

where (x_1, x_2, x_3) are nodal coordinates of the node P. The matrix $\mathbf{Z}_P^{(k)}$ represents six rigid body modes. The effect of MPCs is taken into account in the coarse grid problem using a penalty method. The matrix $\mathbf{K}_\mathrm{C}$ is modified as follows:

$$\mathbf{K}_\mathrm{C} = \mathbf{R}^\mathrm{T}\left(\mathbf{S} + \alpha\,\mathbf{B}\mathbf{B}^\mathrm{T}\right)\mathbf{R} = \mathbf{R}^\mathrm{T}\mathbf{S}\mathbf{R} + \alpha\,\mathbf{R}^\mathrm{T}\mathbf{B}\mathbf{B}^\mathrm{T}\mathbf{R} \qquad (6)$$

where the matrix α is a diagonal matrix that contains penalty constants. The matrix $\mathbf{R}^\mathrm{T}\mathbf{B}$ represents the MPCs for the coarse grid problem. Eq. (5) shows that the coefficients for each MPC in $\mathbf{R}^\mathrm{T}\mathbf{B}$ have very different values when the node P is far from the origin of the coordinate system. In this case, use of the penalty method in Eq. (6) may cause numerical problem. In order to avoid this problem, a local coordinate system is introduced in each subdomain, and a origin is placed in each subdomain. The nodal coordinates of the node P that are used in Eq. (5) are defined using the local coordinate system.

Fig. 1 shows a mesh of a steel frame, which is made of hexahedral elements. A small slab is attached to the frame using MPCs. Two different origins (points A and B) are considered. The model using the origin A is called model SC, and that using the origin B is called model CC. Linear structural analysis is conducted for each of these two models. Fig. 2 shows the convergense properties of the CG method. In this figure, “Fixed Origin” means the origin is fixed at the point A or B to define $\mathbf{Z}_P^{(k)}$. “Moving Origin” means the local coodinate system in each subdomain is used to define $\mathbf{Z}_P^{(k)}$. In the case of “Fixed Origin”, the convergence property is affected by the position of the origin. On the other hand, the same convergence propertis are obtained for the models SC and CC when the “Moving Origin” is used.

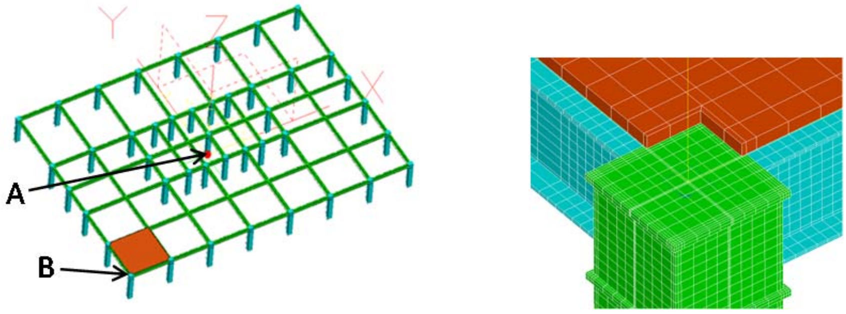

Figure 1. Finite element mesh of a steel frame with a small slab connected by MPCs

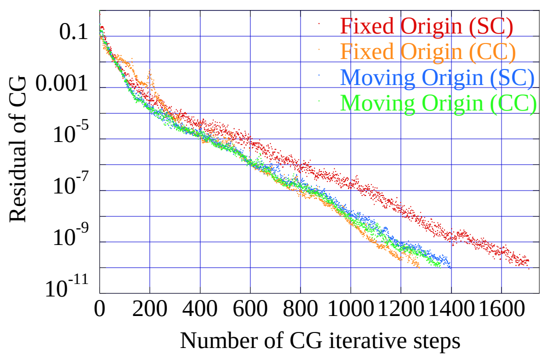

Figure 2. Convergense of CG method preconditioned by BDD method

REFERENCES

[1] T. Miyamura, Incorporation of multipoint constraints into the balancing domain decomposition method and its parallel implementation. *IJNME*, Vol. **69**, 326-346, 2007.