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A TAYLOR MESHLESS METHOD FOR HYPERELASTICITY

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A meshless method based on Taylor series approximation is extended in the non-linear range. This technique was introduced in [1]. The proposed technique uses high degree polynomial shape functions and the Partial Differential Equations are solved in the domain by using Taylor series approximations. Thus one has only to discretize the boundary by a cloud of points. A least square method is used to apply boundary conditions, which permits to stabilize the collocation method. If necessary, several Taylor series can be computed in several subdomains and they can be coupled by methods based on Lagrange multipliers. The accuracy and the efficiency of this technique were proved in [1] [2] [3]. It was shown that the technique converges exponentially with the degree (p-convergence) and with much less degrees of freedom than classical discretization methods: for instance, in a benchmark discussed in [2], about hundred dof's were sufficient to recover a solution requiring 40 000 dof's with P1 finite elements.

In this talk, the application field of this Taylor Meshless Method (TMM) is extended in the non-linear range, especially in hyperelasticity. Because TMM relies on a combination of polynomial solutions, it can be only applied to linear equations. According to an idea of [4][5], the non-linear system is first transformed in a sequence of linear equations, here via the Newton-Raphson classical method. Next the resulting linearized equations are solved by combining Taylor series inside the domain and least-square collocation for the boundary conditions. The key point is the computation of all the polynomials that solve the linearized hyperelastic problem. Some parts of this computation are performed by using the techniques of Automatic Differentiation [6]. A numerical example is presented below, where a rectangle undergoes a deformation up to 100%. This result is obtained after only 8 Newton-Raphson steps and this very accurate result needs only a degree 8, i.e. 34 degrees of freedom at each step. Likely the presented method could be extended for many non-linear equations in various fields of physics and engineering.

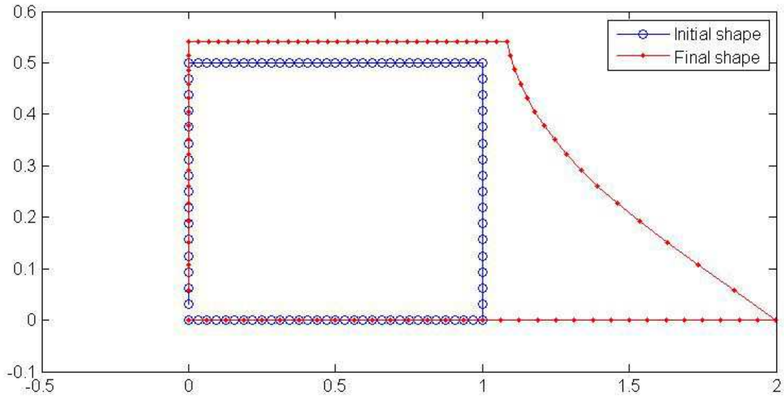

Figure 1. Large deformations of a hyperelastic rectangle are computed after 8 Newton-Raphson steps. A maximal error of 1.5 10^{-5} is obtained with polynomials of degrees 8.

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