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SPACE-TIME SEPARATED REPRESENTATION FOR SOLVING NAVIER-STOKES EQUATIONS

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The Proper Generalized Decomposition (PGD) method has been used to solve many kinds of problem in multi-dimensional spaces: quantum chemistry, kinetic theory description of complex fluid and chemical master equations, etc...The interested reader can refer to [1] for a recent review in the context of computational rheology. The present work describes a first implementation of the PGD formulation into a general unstructured finite-volume unsteady Navier-Stokes solver dedicated to viscous ship hydrodynamics (ISIS-CFD [2]).

1 PGD decomposition for the time-space domain

The following separated decomposition for the pressure and velocity fields is injected into the unsteady Navier-Stokes equations:

$$\begin{aligned} u(x,t) &\approx \textstyle\sum_{i=1}^{N} X_i^u(x)T_i(t) + X^uT \\ v(x,t) &\approx \textstyle\sum_{i=1}^{N} X_i^v(x)T_i(t) + X^vT \\ p(x,t) &\approx \textstyle\sum_{i=1}^{N} X_i^p(x)T_i(t) + X^pT \end{aligned} \tag{1}$$

which leads to coupled non-linear steady spatial and temporal equations replacing the traditional incremental approach. The spatial modes are determined by using the fully coupled approach implemented in ISIS-CFD [3]. This steady formulation is based on the iterative solution of a saddle point linear system coupling together momentum and mass conservation equation transformed into a pressure equation. Steady spatial 2D or 3D partial differential equations and temporal ordinary differential equation are coupled with the help of a fixed-point algorithm. The discretisation based on an unstructured finite-volume formulation is described in [2]. The result shows that the new computational algorithm reduces drastically the computational time needed for the simulation of complex unsteady flows. This non-incremental solution strategy opens a new perspective towards radical evolutions of the computational methods for the simulation of unsteady viscous flows.

2 Results - CPU time comparison

The laminar viscous flow inside a lid-driven cavity with an oscillating upper wall is used to compare the performances of the PGD formulation with respect to the traditional incremental algorithm. Fig.(1) shows the CPU time used by both methods, illustrating the strong reduction implied by the PGD formulation.

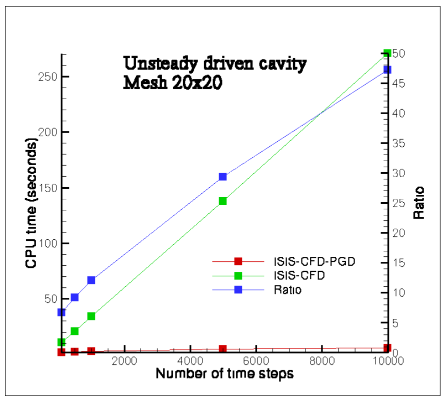

Figure 1: CPU time comparisons for the lid-driven cavity

3 Conclusions

The PGD formulation applied to the unstructured finite-volume industrial Navier-Stokes solver ISIS-CFD, has been assessed successfully on a first academic test case. An impressive reduction of the computational time has been observed on this first application, illustrating promising perspectives.

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