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Detection of bifurcation in a meshless framework

A. Tri†, H. Zahrouni†* and M. Potier-ferry†*
† Laboratoire de Mécanique, Faculté des Sciences Aïn Chok
Université Hassan II, Casablanca, Morocco
{jaliltri@yahoo.fr}
†* Laboratoire d'Étude des Microstructures et de Mécanique des Matériaux, LEM3, CNRS,
Université de Lorraine, Ile du Saulcy, 57045 Metz, France
{hamid.zahrouni, michel.potier-ferry}@univ-lorraine.fr

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ABSTRACT

The originality of this work consists in associating a path following technique, a bifurcation indicators and a meshless technique [1-7]. Here, we are particularly interested in the so called MFS-MPS (Method of Fundamental Solutions-Method of Particular Solution) for the simplicity of its numerical implementation and for robustness to solve partial differential equations with variables coefficients. MFS-MPS permits to discretize PDE's in a meshless framework by combining radial functions [6] and fundamental solutions of a given operator [5, 3]: here we use fundamental solutions of Laplacian and Bi-Laplacian operators.
To show the efficiency of the proposed method, we apply it to the following nonlinear problem (1) on square domain $\Omega = [0,1] \times [0,1]$ where the bifurcation points and the eigenmodes are known analytically.

$$\begin{cases} L(u) + N(u,\lambda) = \lambda \xi f & in\ \Omega \\ B(u) = 0 & on\ \partial\Omega \end{cases} \tag{1}$$

In (1), L and N represent respectively a linear and a nonlinear operator which are considered here as harmonic and bi-harmonic and B is a boundary operator.
A bifurcation point is characterized by the non-invertibility of the tangent operator $L_t(u,\lambda) = L + D_u N(u,\lambda)$. The bifurcation points are found indirectly by seeking the zeros of a scalar function μ, called bifurcation indicator that is computed along the solution path. It is defined from a more or less arbitrary function as follows:

$$\begin{cases} L_t(u,\lambda)\delta u = \mu f_\mu & in\ \Omega \\ B(\delta u) = 0 & on\ \partial\Omega \end{cases} \tag{2}$$

All the unknowns $(u, \lambda, \delta u, \mu)$are searched in the form of a truncated Taylor expansion from a known solution$(u_0, \lambda_0, \delta u_0, 1)$.

$$\big(u(a), \lambda(a), \delta u(a), \mu(a)\big) = \sum_{i=0}^{p} a^i \left(u_i, \lambda_i, \delta u_i, \mu_i\right) \tag{3}$$

where p is the truncation order of the series and 'a' is a control parameter. The resulting linear problems are discretized by MFS-MPS method with the use of multiquadric radial basis functions.
This method has been successfully applied to nonlinear problems with harmonic and bi-harmonic operators. Typical results are presented in Figures 1 and 2, in a case where $L = -\Delta, N = u^2 - \lambda u, B(u) = u$. In figure 1, one sees two response curves with a quasi-bifurcation. As often within Asymptotic Numerical Method, there is a step accumulation close to the bifurcation point. This basic property can be explained by the radius of convergence of the series that coincides with the distance to the bifurcation. Such a step accumulation is an efficient and simple manner to detect a bifurcation point. A second manner to detect a bifurcation point is illustrated in Figure 2: the zero of the indicator defined in (2) yields the position of the bifurcation and the null eigenvector of the tangent operator.

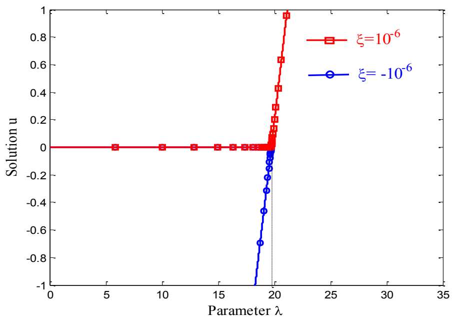

Figure 1: Solution branches by ANM-MFS-MPS

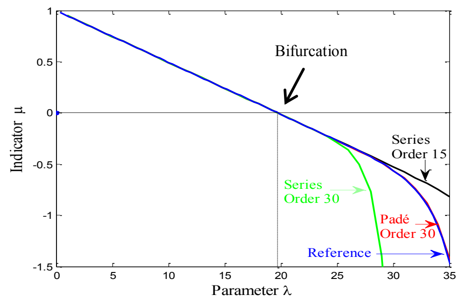

Figure 2: Indicator μ versus λ by ANM-MFS-MPS

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