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# VIBRATIONS OF A STRING WITH ONE NON-MATERIAL BOUNDARY CONDITION

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Problems with non-material boundary conditions are very important in many applications of continuum mechanics. In this context the term “non-material” is related to constraints that cannot be assigned to individual material particles. In literature, there are numerous problems concerning non-material boundary conditions. Without claiming to be complete the following examples can be stated: rolling contact, breaking disks, calenders, belts and band saws. For a survey on current research in this field see [1-2].

Therefore the problem of the vibrating string is chosen as an example to study the characteristics of non-material boundary conditions. The equation of motion for this problem is the one-dimensional wave equation (see [3-4])

$$\frac{\partial^2}{\partial t^2}u(t,x) - c^2\frac{\partial^2}{\partial x^2}u(t,x) = 0\,, \tag{1}$$

herein $c$ denotes the wave propagation speed, $N$ the constant tension and $\varrho$ the linear mass distribution. A shorter operator notation can be found by numbering the arguments of the unknown displacement field $u$

$$\partial_0^2 u - c^2\partial_1^2 u = 0\,, \tag{2}$$

where $\partial_0^2$ is the second time derivative and $\partial_1^2$ the second spatial derivative, respectively. The initial conditions and boundary conditions complete the problem specification for the classical problem

$$u(0,x) = u_0(x) \quad \text{and} \quad \partial_0 u(0,x) = u_1(x) \quad \text{for} \quad 0 \le x \le l\,, \tag{3}$$

$$u(t,0) = 0 \quad \text{and} \quad u(t,l) = 0 \quad \text{for} \quad t \ge 0\,. \tag{4}$$

Analogously the problem of a vibrating string with one non-material boundary condition can be formulated (see Fig. 1). With the same assumptions the equation of motion of this problem is also the one-dimensional wave equation. The only differences in the problem specification are the boundary conditions

$$u(t,0) = 0 \quad \text{and} \quad u(t,V(t)+l) = 0 \quad \text{for} \quad t \ge 0\,, \tag{5}$$

where the distance of motion of the bearing on the right hand side is represented by the integral

$$V(t) = \int_0^t v(\bar{t}\,)\,\mathrm{d}\bar{t}\,. \tag{6}$$

An integral approach can be derived to find an analytical solution for the specified one-dimensional problem.

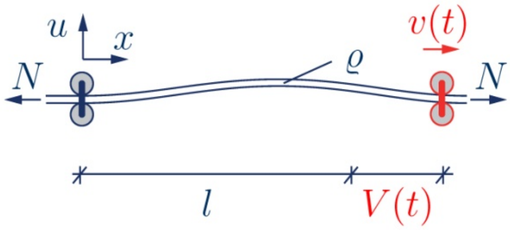


Fig. 1 Problem specification, vibrating string with one non-material boundary condition

Experiments are performed to verify the derived solution strategy. The test object is a musical instrument called monochord, i.e. a pretensioned string mounted on a resonator. Within the experimental setup, the monochord is equipped with a microphone and a movable wire pulley bearing (see Fig. 2). Comparisons between experiments and simulations show a quite good agreement.


Fig. 2 Scheme of the experimental setup for a monochord with one non-material boundary condition

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