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Reduced numerical methods applied to thermoconvective problems

Henar Herrero[1], Francisco Pla[2] and Yvon Maday[3]

[1] *Universidad de Castilla-La Mancha, Dpto. de Matemáticas, 13071 Ciudad Real, Spain, Henar.Herrero@uclm.es*

[2] *Universidad de Castilla-La Mancha, Dpto. de Matemáticas, 13071 Ciudad Real, Spain, Francisco.Pla@uclm.es*

[3] *Université Pierre et Marie Curie, Laboratoire Jaques-Louis Lions, Paris, France, maday@ann.jussieu.fr*

The reduced basis approximation is a discretization method that can be implemented for solving of parameter-dependent problems $\mathcal{P}(\phi(\mu),\,\mu)=0$ with parameter μ in cases of many queries. This method consists in approximating the solution $\phi(\mu)$ of $\mathcal{P}(\phi(\mu),\,\mu)=0$ by a linear combination of *appropriate* preliminary computed solutions $\phi(\mu_i)$ with $i=1,\,2,\,...N$ such that μ_i are parameters chosen by an iterative procedure using the *kolmogorov n-width* measures [2, 4].

In this work [1] it is applied to a two dimensional Rayleigh-Bénard problem with constant viscosity that depends on the Rayleigh number, $\mathcal{P}\left(\phi(R),\,R\right)=\vec{0}$, as follows:

$$0 = \nabla\cdot\vec{v}, \quad \text{in } \Omega, \tag{1}$$

$$\frac{1}{Pr}\left(\partial_t\vec{v}+\vec{v}\cdot\nabla\vec{v}\right) = R\,\theta\vec{e_3}-\nabla P+\nu\Delta\vec{v}, \quad \text{in } \Omega, \tag{2}$$

$$\partial_t\theta+\vec{v}\cdot\nabla\theta = \Delta\theta, \quad \text{in } \Omega, \tag{3}$$

where $\Omega=[0,\Gamma]\times[0,1]$, $\phi=(\vec{v},\,\theta,\,P)$ and $\vec{v}$ is the velocity vector field, θ is the temperature field, P is the pressure, $\vec{e}_3$ is the unitary vector in the vertical direction, R is the Rayleigh number and Pr the Prandtl number.

For each fixed value of the aspect ratio Γ, multiple steady solutions can be found for different Rayleigh numbers and stable solutions coexist at the same values of external physical parameters [3]. The reduced basis method permits to speed up the computations of these solutions at any value of the Rayleigh number chosen in a fixed interval associated with a single bifurcation branch while maintaining accuracy.

The problem is numerically solved by the Galerkin variational formulation using the Legendre Gauss-Lobatto quadrature formulas together with the reduced basis $\{\phi(R_i), i=1,\,2,\,...N\}$ such that $\phi(R)\sim\sum_{i=1}^{N}\lambda_i\phi(R_i)$.

The bifurcations can be captured with this method. It is useful for bifurcation continuation purposes.

References

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