

Acceleration by RK/Implicit smoother for coupled Navier-Stokes and heat transfer

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Key Words: *acceleration Navier-Stokes, heat transfer.*

Many important scientific and engineering problems involve the coupling of fluid flow and heat transfer in solids. This includes modeling of heat exchangers, cooling of turbines blades and heat transfer in rockets engines. In this paper the conjugate heat-flow code is described and validated for simple cases and compared to analytic solutions. The problems of the flow over a heated plate and conjugate steady/unsteady flow in converging diverging nozzle are also presented.

The scheme used for the fluid domain is a finite volume, with the AUSM+UP scheme for the spatial discretization and a dual time step approach for the unsteady problem. For the pseudo-time integration we use a Runge-Kutta scheme with an implicit smoother. In the solid domain, we use an implicit ADI scheme. To accelerate the convergence we use the RK/Implicit smoother [1,2]. This enables the use of time steps beyond the CFL condition. The RK/Implicit smoother method was first developed for viscous and inviscid flow and was extended to multi-equation turbulent reactive flow and two-phase with Eulerian dispersed phase. In this work we will present an improved dealing of the physical time integration in the context of the RK/implicit smoother scheme.

The problem contains two regions: a fluid region adjacent to a solid region. The governing system in the fluid region is the Navier-Stokes equations. The flow is considered to be turbulent and the model equations for the turbulence is the $k-\omega$ SST model for two equation model or the Spalart-Allmaras for one equation model. The governing equation in the solid region is the heat equation. In addition, an appropriate solid-fluid interface condition is required. This interface conditions should preserve the continuity of the temperature and heat flux.

The first example is for Riemann problem (Sod's shock tube). The purpose is to validate the dual time algorithm. The convergence for a series of CFL numbers is presented in figure 1. As can be seen, the convergence becomes better as the CFL increases, until reaching an asymptotic rate of convergence.

In the next example, supersonic, steady state flow inside a cooled axisymmetric convergent-divergent nozzle is calculated. The analysis is based on the experimental data reported by Back et al. [3]. Temperature contours in the fluid and inside the wall are shown in figures 2. Comparison of the calculated heat flux coefficient distribution is presented in figure 3. Very good agreement between the computational result and the experimental data obtained.

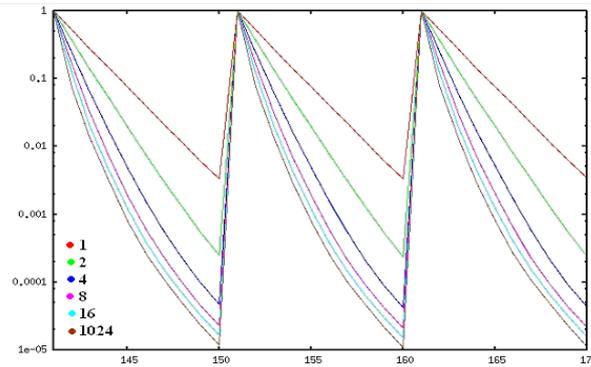


Figure 1: convergence history for series of CFL numbers.

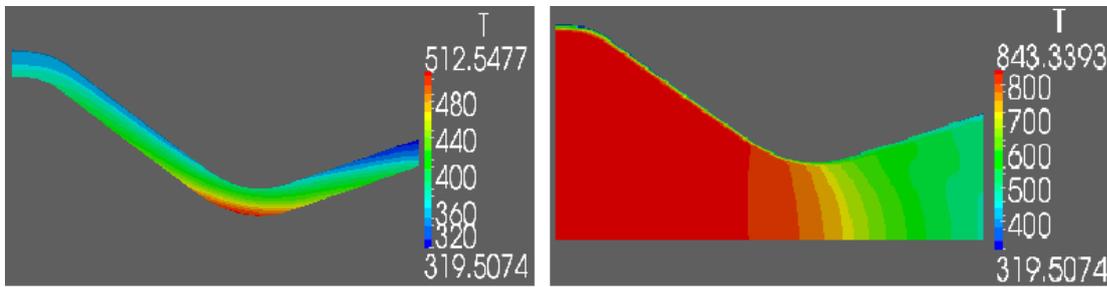


Figure 2: temperature contours inside the solid wall (left) and in the gas (right)

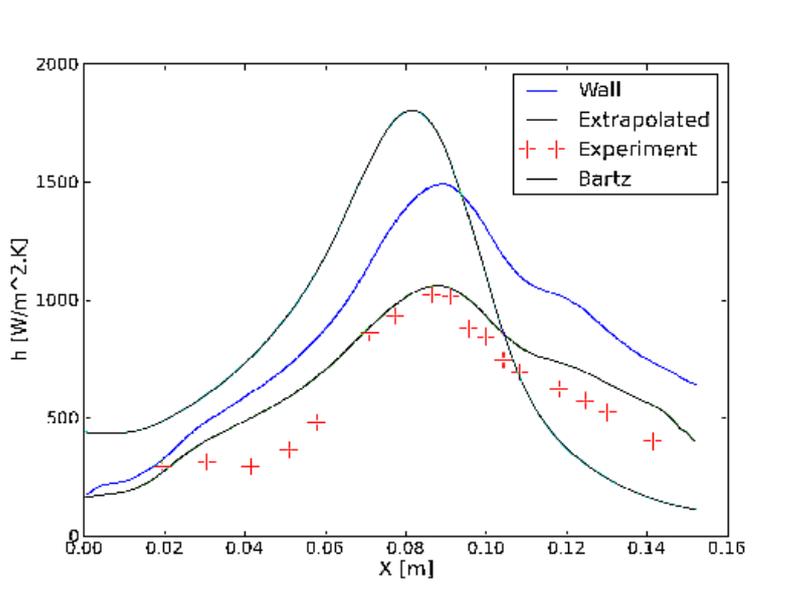


Figure 3: Comparison of calculated heat flux coefficient distribution along nozzle wall.

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