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VOLUME CONSERVATION OF 3D SURFACE TRIANGULAR MESH SMOOTHING

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Laplacian and Gaussian smoothings are commonly used techniques to improve quality of (not only) finite element meshes. Their popularity reside in simple and efficient implementation and good performance (especially) on isotropic simplicial meshes. While their application to 3D and planar 2D meshes is straightforward, use for 2.5D meshes on curved surfaces suffers from (usually undesirable) shrinkage as the smoothed node is attracted toward center of curvature. This can be efficiently prevented by returning the smoothed nodes back onto the surface. This is quite easy if analytical (typically parametric) description of the surface is available. But this may be not always the case, for example when handling discretely described surfaces.

In past, different approaches for volume preserving smoothing were developed. Some algorithms [1, 2, 3] are based on the local volume control using vertex balance procedure. Other techniques [4, 5] combine the shrinking phase with an expansion phase during each smoothing cycle. While the approach in [4], presented as a low pass filter removing high curvature variations, is utilising two Gaussian smoothing subcycles (within each smoothing cycle), one with positive weights, the other one with negative weights, authors of [5] combine (within each smoothing cycle) Laplacian smoothing of points with Gaussian smoothing applied to differences between points before and after the Laplacian smoothing. In [6], a shrinkage-free interpolatory smoothing, based on the assumption that centers of all (arbitrarily polygonal) facets of the mesh are located on the anticipated geometry of the surface, is introduced. The smoothing scheme is applied to a larger stencil of previously subdivided mesh using the $\sqrt{3}$ splitting. A small shrinkage induced due to the linear approximation is then eliminated by the application of the Taubin-like inflation operator.

In this work, a similar approach to that presented in [4] is employed. However, the weights for shrinkage and expansion smoothing subcycles are derived, contrary to [4], where they are determined by Fourier analysis, using the “do-not-harm” concept, the idea of which is to bring a node on the optimal mesh back to its original position after both subcycles. The expansion weight is dependent on the shrinkage weight and is computed directly from the geometrical setting just before the shrinkage subcycle is started. Note that the algorithm was derived for in-plane smoothing of surface meshes. This implies that it is assumed that the nodes of the original mesh are located on the surface. The application of the algorithm to meshes with random noise in the sense of out-of-surface deviation is under investigation. While the application of the proposed approach to a closed surface is trivial, its processing on open surfaces is more complicated. In order to make the expansion phase working correctly, the shrinkage phase must be properly represented also on the boundary nodes. Since all sharp features are assumed to be topologically explicitly present in surface description, no other special treatment (as sharpness dependent filters [7], for example) is necessary. The actual smoothing is performed hierarchically. Firstly the boundary curves are smoothed, with special treatment at boundary vertices, and the individual surfaces are smoothed, again with special treatment at boundary nodes. The performance and capabilities of the presented methodology are demonstrated on several examples.

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