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EFFECTIVE BOUNDARY CONDITIONS FOR COMPRESSIBLE FLOWS OVER A ROUGH SURFACE

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Domains with microscopic rough boundaries frequently arise in applications in engineering. For instance, space shuttles are often covered with tiles, while small air injecting nozzles are used over wings of aircrafts to reduce the drag [7]. Examples can also be found in nature, e.g. the skin of sharks [4], and in everyday life, e.g. golf balls.

The challenge inherent in the numerical simulation of such problems is the high resolution that is needed to resolve the roughness. In general, the computational costs are prohibitively high and a direct numerical simulation is not feasible. To deal with this type of problems we thus need concepts that allow to quantify the influence of small scale effects on the resolved large scale effects *without* resolving small scale structures.

A possible approach for the derivation of an appropriate upscaling strategy is to smooth artificially the boundary and solve the flow equations in the artificial smooth domain, cf. [1, 5], applying a *homogenization technique* [2, 6, 8]. Of course, this will introduce a significant error because micro-scale effects due to the roughness are discarded in this *zeroth order solution*. Therefore it has to be corrected by an appropriate correction term that depends both on macro-scale *and* micro-scale variables. Plugging the modified solution into the original problem, the so-called *cell problem* can be derived by means of an asymptotic expansion: this auxiliary problem is typically much simpler and it is defined only on the micro-scale. In addition, we obtain also a correction for the boundary conditions on the artificial smooth surface via avareged quantities of the solution of the cell problem. These are referred to as *effective boundary conditions* or *Navier wall law* [3] and can be considered the upscaling model. Finally the effective problem can be solved on the smooth domain with the effective boundary conditions.

The aim of this presentation is to extend the ideas of Achdou et al. [1] to the *multidimensional, compressible* flow regime for *high* Reynolds number. The underlying dynamics is much more complicated: transport effects are dominating dissipative effects due to viscosity and heat conduction. Moreover, boundary layers are much thinner and therefore more sensitive to perturbations. As a consequence some simplifications adopted in [1] cannot be applied here and new ideas must be taken into account. In particular, for compressible flows the solution of the zeroth order approximation will enter the cell problem, so we must deal with a coupling between micro and macro scales. Moreover an adaptive multi-scale algorithm must be adopted to deal with the intrinsic ill-conditioning of the effective problem.

For proof of concept we investigate the laminar flow over a flat plate with partially embedded periodic riblets. Of particular interest is the skin friction coefficient that serves as a measure for the quality of the effective model compared to direct numerical simulations performed on the rough domain.

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