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NUMERICAL METHODS FOR THE OPTIMIZATION OF NONLINEAR RESIDUAL-BASED SUBGRID-SCALE MODELS USING THE VARIATIONAL GERMANO IDENTITY

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The Variational Germano Identity [4][1] is used to optimize the coefficients of residual-based subgrid-scale models that arise from the application of a Variational Multiscale Method [3][2]. It is demonstrated that numerical iterative methods can be used to solve the Germano relations to obtain values for the parameters of subgrid-scale models that are nonlinear in their coefficients. Specifically, the Newton-Raphson method is employed. A least-squares minimization formulation of the Germano Identity is developed to resolve issues that occur when the identity is positive and negative over different regions of the domain. In this case a Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm is used to solve the minimization problem.

The developed method is applied to the one-dimensional unsteady forced Burgers' equation and the two-dimensional steady Stokes' equations. It is shown that the Newton-Raphson method and BFGS algorithm generally solve, or minimize, the Germano relations in a relatively small number of iterations. The optimized subgrid-scale models are shown to outperform standard SGS models with respect to an L_2 error. Additionally, nonlinear SGS models tend to achieve lower L_2 errors than the linear models. Ongoing work is aimed at extending the current method to the three-dimensional incompressible Navier-Stokes equations.

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