11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

SMOOTHED NONLINEAR COMPLEMENTARITY FUNCTIONS FOR ELASTO-PLASTIC FRICTIONAL CONTACT AT FINITE STRAINS

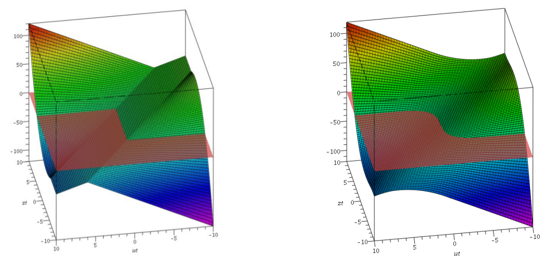
Alexander Seitz*[1], Alexander Popp[1] and Wolfgang A. Wall[1]

[1] Institute for Computational Mechanics, Technische Universität München, Boltzmannstr. 15, D–85748 Garching b. München, Germany, seitz@lnm.mw.tum.de, http://www.lnm.mw.tum.de

Key words: *finite strain plasticity, frictional contact, mortar finite element methods, dual Lagrange multipliers, semi-smooth Newton methods.*

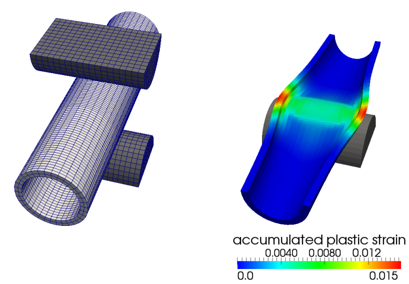
Throughout the past decade, a lot of research in computational contact mechanics has been focusing on efficient and robust solution schemes, such as dual Lagrange-multiplier methods based on a mortar finite element discretization together with a semi-smooth Newton method for the constraint enforcement [4]. While solution techniques have been widely discussed for contact mechanics, the focus in computational plasticity has mainly been on sophisticated yield and hardening models representing the physical material behavior more and more precisely. The actual enforcement of plastic constraints is commonly still performed in a local Gauss-point-wise manner, e.g. trough radial return-mapping methods [5]. The drawback of this procedure, however, is that the constraints need to be fulfilled at each iteration step in the nonlinear solution process and not only at convergence, which may result in a rather small radius of convergence of Newton's method.

In this contribution, we present a semi-smooth Newton - active set strategy for both the frictional contact and plastic material constraints. In computational contact mechanics, such strategies have become quite sophisticated and have been applied successfully to finite deformations, e.g. in [2, 4]. For small strain plasticity, semi-smooth Newton methods have already been proposed in [1] and further elaborated in [3]. Analogously to the contact case, we reformulate the plastic constraints at each quadrature point as a tensor-valued semi-smooth nonlinear complementarity (NCP) function by exploiting the similarity between von–Mises plasticity and isotropic friction. Therein, the plastic kinematics is based on a multiplicative split of the deformation gradient. Moreover, any isotropic hyperelastic law can be used to describe the elastic material behavior. By doing so, and introducing the discrete plastic flow as additional unknowns, all nonlinearities, including nonlinear kinematics, elasticity, plasticity and frictional contact can be dealt with in one single Newton-type iteration. Thereby, the fulfillment of the plastic constraints is only enforced at convergence, whilst not required in the pre-asymptotic range. This



(a) Semi-smooth (b) Smoothed

Figure 1: Semi-smooth and smoothed NCP function for friction and von–Mises plasticity.



(a) Initial (b) Deformed

Figure 2: Initial mesh and deformed configuration of a squeezed tube.

less restrictive formulation significantly improves the robustness of the overall nonlinear solution procedure without compromising on accuracy. A discontinuous interpolation of the plastic deformation allows for an elimination of the additional unknowns, such that the resulting system of equations only consists of displacement degrees of freedom.

A further stabilization of the active set algorithm is achieved by applying a local smoothing procedure to the NCP functions in the pre-asymptotic range. Figure 1 illustrates the regular as well as the smoothed complementarity functions for frictional contact or, equivalently, for von–Mises plasticity. To ensure accurate results at convergence, the smoothing is successively reduced over the nonlinear iteration process. Besides a significant stabilization of the active set search, this smoothing of the semi-smooth NCP-functions also allows for the application of well-known methods of smooth optimization such as line-seach methods. In conclusion, the presented method offers a very robust and efficent way to solve strongly nonlinear problems of elasto-plastic contact in the presence of large deformations as exemplarily illustrated in Figure 2.

REFERENCES

[1] P. Christensen. A nonsmooth Newton method for elastoplastic problems. *Computer Methods in Applied Mechanics and Engineering*, Vol. **191**, 1189–1219, 2002.

[2] M. Gitterle, A. Popp, M.W. Gee and W.A. Wall. Finite deformation frictional mortar contact using a semi-smooth Newton method with consistent linearization. *International Journal for Numerical Methods in Engineering*, Vol. **84**, 543–571, 2010.

[3] C. Hager and B. Wohlmuth. Nonlinear complementarity functions for plasticity problems with frictional contact. *Computer Methods in Applied Mechanics and Engineering*, Vol. **198**, 3411–3427, 2009.

[4] A. Popp, M. Gitterle, M.W. Gee and W.A. Wall. A dual mortar approach for 3D finite deformation contact with consistent linearization. *International Journal for Numerical Methods in Engineering*, Vol. **83**, 1428–1465, 2010.

[5] E.A. de Souza Neto, D. Perić and D.R.J. Owen. *Computational methods for plasticity: Theory and applications*. Wiley, 2008.