## ON MODELS FOR INTERFACES: THEORY AND COMPUTATIONAL RESULTS

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## 1 Introduction

Interface models are widely used for structural analyses in several fields of engineering applications. They are adopted to simulate different structural situations as, for instance, to reproduce the crack evolution in a body according to the cohesive fracture mechanics, to study the delamination process for composite laminates, to simulate the presence of strain localization problems or to model the bond between two or more bodies.

There exist various interface models in literature, according to the mechanical properties of the adhesive. In this paper, we will consider two of the principal sets of model, within the linear elastic framework. The first one consists in considering that the rigidity of the adhesive is lower than those of the adherents. Then, using an asymptotic theory where the small parameter is both the thickness and the elastic coefficients of the adhesive, leads to an interface law at order 0 in terms on the small parameter where the interface behavior is modeled by linear springs. This model is usually called soft interface law and is widely treated in literature, both theoretically and numerically.

On the contrary, when the rigidity of the adhesive is of the same order than those of the adherents, and it is no longer a small parameter, then, at order 0, the interface behaves as a perfect glue, where the jumps in the displacement and in the constraint vanish. Even if this model, called hard interface, is treated in literature, it is less rich than the previous

one. In order to enrich the modeling, it is necessary to consider further terms in the asymptotic expansion. It can be shown that, at order 1, the jumps in the displacement and in the constraint across the interface take the form

$$[u^1] = f(u^0, \sigma^0 n)$$
  
 $[\sigma^1] = g(u^0, \sigma^0 n)$ 

where  $[\cdot]$  represents the jump along the interface, f and g are given functions that depend on the mechanical properties of the adhesive. In particular, a membrane effect due to the strong rigidity of the adherent is taken into account in this modeling, since the jump in the constraint depends on the interface curvature.

In order to compare the two models of interface, a soft interface law at order 1 is also developed. This law is very similar to that of the hard interface law, but without membrane effect.

Moreover, an original method based on the asymptotic expansion of the mechanical energy of the system is developed to obtain the various interface laws.

The models presented in this study will be compared accordingly to the thickness and the relative rigidity of the adhesive and the adherents.

## 2 Numerical results

A special attention will be paid on the development of numerical procedures to solve the problems of both soft and hard interfaces at order 1, that are non standard, because classical weak formulations can not be easily derived from the governing equations. Then, some numerical examples will be presented in order to compare the various approaches (soft and hard interface models at order 0 and 1) to the initial problem of three phases elastic bodies where the adhesive possesses a finite thickness  $\varepsilon$ .

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