

RECURSIVE BLOCK PRECONDITIONERS FOR MULTIPHYSICS PROBLEMS: APPLICATION TO INCOMPRESSIBLE MHD

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Incompressible magnetohydrodynamics model the flow of an electrically charged fluid under an external electromagnetic field. This system of partial differential equations is strongly coupled and highly nonlinear for real cases of interest. Therefore, the use of fully implicit time integration methods are very convenient in order to be able to capture the several physical scales of the problem. Moreover, realistic simulations can reach up to $10^7 - 10^9$ degrees of freedom. Therefore, solving the resulting multiphysics linear systems of equations is a very challenging task. Our approach to tackle this problem is the development of recursive block preconditioners to increase the convergence rate of iterative Krylov solvers.

A new family of recursive block LU preconditioners is designed for solving a generic multiphysics problem. Next, it is applied for the particular case of incompressible MHD. These preconditioners are obtained after splitting the fully coupled multiphysics system into one-physics problems (one for each physical variable) that can be optimally solved, e.g. using preconditioned domain decomposition algorithms. The main idea consists of reordering the original problem into a 2×2 block matrix and consider an incomplete LU factorization as preconditioner by approximating the corresponding Schur complement. If the resulting block matrices involve more than one variable, another level of LU factorization can be used in a recursive fashion as preconditioners for the inner systems.

A detailed set of numerical experiments shows the good behavior of these preconditioners with respect to scalability and convergence. Moreover, they have proved to be efficient when solving a realistic problem with extreme physical properties such as a TBM for nuclear fusion reactors in a large scale computer.

REFERENCES

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