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A CONJUGATE GRADIENT BASED METHOD FOR FRICTIONAL CONTACT PROBLEMS

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In the simulation of railway vehicles dynamics, the interaction between the vehicles' wheels and rails attracts a lot of interest. It involves the solution of the so-called *contact problems*, concerning the normal and tangential tractions on the inter-surface. The formulation based on Kalker's variational half-space approach [1] is regarded as an accurate model for contact problems, particularly those involving a rolling contact with friction. The frictional stress arises between two contacting bodies when they are brought into relative motion. The question is to find out which parts of the surfaces are sticking together versus where local relative sliding occurs, and further to find the distribution of frictional stress. Fast solvers are demanded for such problems.

We would like to present a conjugate gradient based method, called "TangCG", which is incorporated with active set strategy. It is a further study based on the BCCG algorithm which was proposed for normal contact problems [2]. Moreover, fast Fourier transform (FFT) technique [3] is used to accelerate the matrix-vector products encountered in the algorithm. These components are depicted in Fig.1(a).

One significant difference with the conventional solvers lies in the change of unknowns in slip area, where the magnitude of tractions reaches the traction bound. A polar coordinate is used for each sliding element, see Fig.1(b). The traction $\mathbf{p}$ moves on a circle with radius

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being equal to the traction bound g, and is uniquely determined by the angle α. In slip area we solve for the angle α, which must equals to π plus the angle of resulting slip $\mathbf{s}$, since the directions of tractions and corresponding slip are opposite. Linearization technique is employed for some necessary approximation in the algorithm.

Table 1 shows the results for a Cattaneo problem with different situations of slip and different discretization. It implies the efficiency and robustness of our method.

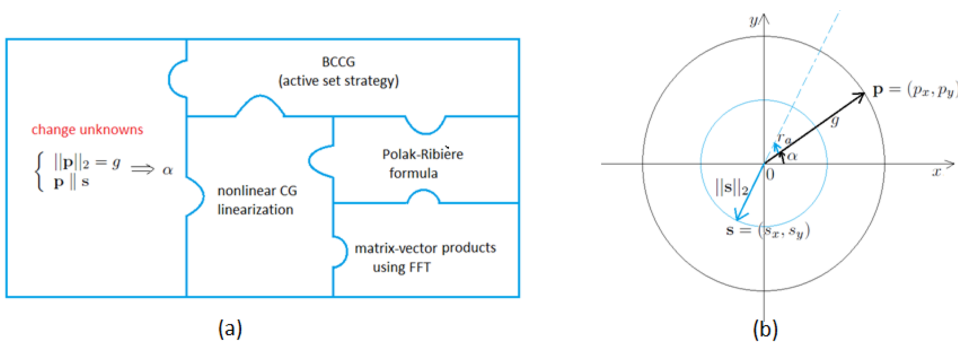

Figure 1: (a) The components of TangCG algorithm. (b) The new formulation on one sliding element of TangCG.

Discretization	5×5	30×30	60×60	90×90
no slip	7 (0.02)	14 (0.06)	19 (0.31)	21 (0.76)
%20 slip	11 (0.03)	30 (0.12)	35 (0.58)	53 (1.79)
%80 slip	11 (0.03)	26 (0.11)	29 (0.47)	36 (1.45)
full slip	8 (0.02)	19 (0.09)	24 (0.37)	24 (0.95)

Table 1: The iteration numbers with CPU time (seconds) in brackets for different discretizations on different slip cases.

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