

# DISSIPATION-BASED APPROACH AND ROBUST INTEGRATION ALGORITHM FOR 3D PHENOMENOLOGICAL SMA CONSTITUTIVE MODELS

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Shape memory alloys (SMA) are materials which, after being subjected to a severe apparently plastic deformation, can recover their original shape if subjected to an appropriate temperature increase. Such unique mechanical behavior is associated with stress-induced phase transformations from twinned to detwinned martensite (during the plastic deformation) and from detwinned martensite to austenite (during shape recovery). The former characteristic is referred to as superelasticity, the latter as shape memory effect. Shape memory alloys can be engineered at thermo-chemical level according to prescribed specifications and are utilized in a wide set of engineering applications.

One of the most attractive approaches for the thermo-mechanical modeling of SMA is thermodynamics with internal variables [1], a consolidated framework for the development of constitutive models consistent with the fundamental principles of thermodynamics of irreversible processes. The main feature of this approach is to introduce an appropriate energy potential  $\psi$  (Gibbs free energy or Helmholtz free energy) depending on *internal variables*, usually an inelastic macroscopic strain tensor  $\mathbf{e}^{\text{tr}}$  (*transformation strain*), and on observable control variables, usually a total strain tensor  $\boldsymbol{\varepsilon}$  and the temperature  $T$ . Classically, constitutive equations are derived writing *state equations*, which define entities conjugate to the control variables and to the internal variables, together with an evolution equation for the transformation strain rate  $\dot{\mathbf{e}}^{\text{tr}}$ . The latter is usually postulated as a flow rule associated to some plasticity-like yield function, or *transformation-function*, defining the elastic domain. This type of model has attracted great attention by researchers over the years, and is the typical constitutive solver in most engineering FEM codes [2].

The present work focuses on the SMA constitutive model format originally proposed

by Souza et al. [3] and subsequently developed by Auricchio and Petrini [4], where a Helmholtz free energy function  $\psi(\boldsymbol{\varepsilon}, T, \mathbf{e}^{\text{tr}})$  is considered, split into elastic, transformation, and chemical contributions. The transformation function is taken as the classical von-Mises or Prager-Lode type, respectively, in the first and second cited paper. The solution procedure of those instances implements a backward Euler scheme for the integration of the evolution equation, coupled with a branch detection scheme, distinguishing between three different material phases. The update of transformation strain and stress tensors is carried out using a return map strategy.

More recently, Mielke and Stefanelli have recast the aforementioned format in the rigorous convex analytic setting of the so-called *energetic formulations* of rate-independent processes [5]. The constitutive model under consideration admits the form of a non-smooth nonlinear differential inclusion:

$$\mathbf{0} \in \partial\psi(\boldsymbol{\varepsilon}, T, \mathbf{e}^{\text{tr}}) + \partial D(\dot{\mathbf{e}}^{\text{tr}}) \quad (1)$$

where  $D(\dot{\mathbf{e}}^{\text{tr}})$  is the dissipation potential. The present paper is based on the previous formulation, and on the isotropy assumption on the transformation function. The solution procedure adopts a backward Euler integration method for rate equations. The so called Haigh-Westergaard stress space parametrization is applied to efficiently compute the dissipation potential and its derivatives, thus allowing for an efficient Newton-Raphson solution for the transformation strain. The present work aims at providing the following results: (i) A detailed understanding of singularities inherent to the variational formulation (1), (ii) A robust solution procedure capable of treating the above singularities, (iii) A generalization to more sophisticated transformation potential forms, (iv) An efficient user material model subroutine for FEM implementation, coded in the innovative parallel computing Nvidia® CUDA™ paradigm. Numerical tests on a single integration point, and 3D FEM simulations are provided to prove such points.

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