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EXTENDED HYBRIDIZABLE DISCONTINUOUS GALERKIN (X-HDG) FOR BIMATERIAL PROBLEMS ♣

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As the range of phenomena that need to be simulated in engineering practice broadens, the limitations of conventional computational methods have become apparent. There are many problems of industrial and academic interest which cannot be easily treated with classical methods, requiring the development of new advanced discretization techniques.

In particular, high-order Discontinous Galerkin methods are nowadays very popular in the CFD community. DG methods inherit the advantages of Finite Volume methods (stability through numerical fluxes, local conservation, etc) but they allow for the use of high-order approximations with a straight-forward implementation of p-adaptivity. Among all DG methods, the novel Hybridizable Discontinuous Galerkin method (HDG [1]) has proved outstanding efficiency, close to the computational efficiency of high-order Continuous Finite Elements (CFE) even for elliptic problems [2]. The hybridization process in HDG allows reducing the degrees of freedom to the nodes in the element faces (sides in 2D), similarly to static condensation in CFE. Moreover, a simple post-process of the solution provides a superconvergent approximation that can be used to define a simple and efficient error estimator, which can be used to drive an adaptivity process [3].

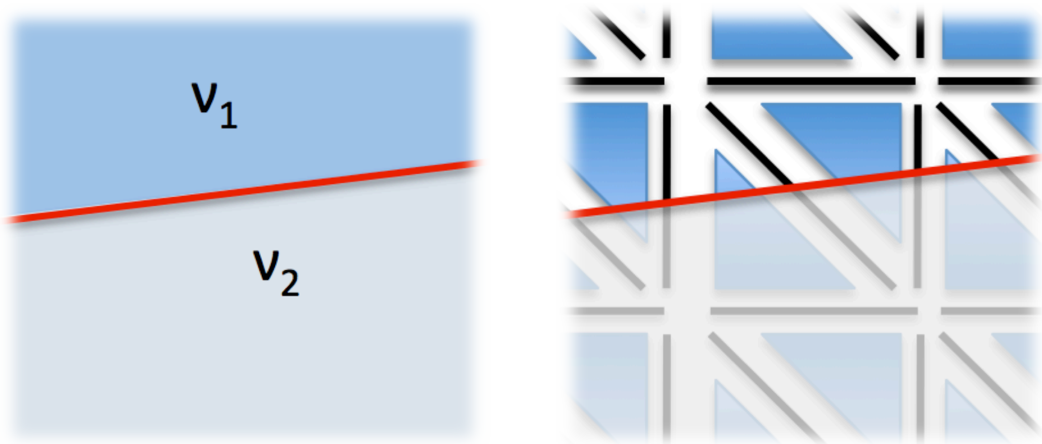

Figure 1. Representation of a bimaterial problem and HDG discretization: elements (blue) and sides (black) are cut by the material interface (red)

On the other hand, the eXtended Finite Element method (X-FEM [4]) is a clever strategy to treat, for instance, the discontinuities arising at interfaces in bimaterial problems. Interfaces are usually represented as the 0-level set of a signed distance function, the solution is enriched to represent weak or strong discontinuities across interfaces, and numerical integration is adapted to take care of the discontinuous approximation inside elements. X-FEM has recently been successfully applied with high-order approximations in the context of CFE [5,6].

This work proposes a formulation for the efficient solution of bimaterial problems, based on these two advanced discretization techniques: the eXtended Hybridizable Discontinuous Galerkin (X-HDG) method. The X-FEM philosophy is introduced in an HDG formulation. The solution is enriched with Heaviside functions and, in the case of weak discontinuities, continuity is weakly imposed, emulating the imposition of continuity across element boundaries in standard HDG. The formulation is first developed for elliptic problems and then extended to bimaterial flow problems.

The proposed methodology aims to take advantage of the HDG superconvergence properties in the context of bimaterial flow problems. In particular, the optimal convergence of the solution gradient (with same order as the primal solution) provides a proper representation of fluxes across interfaces.

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