

VARIATIONAL MULTISCALE LARGE EDDY SIMULATION OF TURBULENT INCOMPRESSIBLE FLOWS

S. Badia^{1,2}, R. Codina^{1,2}, O. Colomés¹ and J. Principe^{1,2}

¹ International Center for Numerical Methods in Engineering (CIMNE)
Gran Capitán s/n, 08034, Barcelona, Spain
{sbadia,codina,ocolomes,principe}@cimne.upc.edu

² Universitat Politècnica de Catalunya
Jordi Girona 1-3, Edifici C1, 08034, Barcelona, Spain

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The variational multiscale (VMS) method was introduced in [4] as a framework for the development of stabilization techniques, which aim to overcome numerical difficulties encountered when using the standard Galerkin method, the finite element (FE) counterpart of centered finite differences (FD), namely, the compatibility condition between the velocity and pressure FE spaces, which require the use of staggered grid in the FD context, and the nonphysical oscillations that could appear in the convection dominated regime, when the mesh is not fine enough.

The VMS method was then used proposed as an implicit large eddy simulation (LES) technique in [5] although in these early approaches small scales were explicitly solved introducing a Smagorinsky-type dissipative term. As a result, an important fraction of the degrees of freedom are used for the small resolved scales whereas consistency is retained in the large resolved scales only. ILES using a VMS approach with a resolved and a modeled subgrid scale (the setting that permits to recover stabilized formulations) was suggested in [2] and excellent numerical results were obtained in [1]. Compared to explicit LES, the VMS approach does not face difficulties associated with inhomogeneous non-commutative filters in wall-bounded flows and retains numerical consistency in the FE equations up to the interpolation order whereas e.g. the Smagorinsky model introduces a consistency error of order $h^{4/3}$.

The dissipative structure of variational multiscale methods has been analyzed in [6] where it is compared to the physical based LES dissipative structure. Simple LES closures (e.g. Smagorinsky) that could be used as an engineering design tool are purely dissipative whereas VMS methods can predict more complex energy transfers. The way the energy is exchanged between coarse and fine scales depends on the approximation performed on

the fine scale equation. Dynamic and orthogonal subgrid scales introduced in [2, 3] result in a formulation with important numerical properties such as commutativity of space and time discretization or global conservation statements and a dissipative structure that presents the correct behavior in the laminar limit and is able to predict backscatter.

Apart from a discussion of these properties, we present detailed numerical results of the simulation of the decay of homogeneous isotropic turbulence. We study the influence of the different subgrid possibilities, the influence of the algorithmic constants in the stabilization parameters and the computational costs of different approaches. Finally, we compare the results obtained using VMS models against those obtained using classical LES based on filtering and the Smagorinsky closure. We do so using the Galerkin approximation of the Navier Stokes equations with a Taylor-Hood Q2/Q1 interpolation which satisfies the inf-sup condition, relying on the Smagorinsky term to stabilize convection.

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