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DISCRETIZATIONS AND REGULARIZATION MODELS FOR COMPRESSIBLE FLOW THAT PRESERVE THE SKEW-SYMMETRY OF CONVECTIVE TRANSPORT

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It is well-known that flow structures small compared to the mesh spacing may trigger a non-linear convective instability of compressible flow simulations. Therefore, simulation methods for compressible flow often need explicit filtering or artificial dissipation to attain numerical stability. An unpleasant side-effect of these ad hoc stabilization techniques is that they suppress flow phenomena such as turbulence and propagation of acoustic waves. For incompressible flow an alternative road to numerical stability exists; the symmetry-preserving discretization [3]. The symmetry-preserving discretization preserves the skew-symmetry of the convective terms at the discrete level. This skew-symmetry prevents the non-physical creation of discrete kinetic energy through convective transport, and thereby eliminates the corresponding numerical instability. A large-eddy-simulation model that follows the same line of thought is the symmetry-preserving regularization model for incompressible flow [4]. Symmetry-preserving regularization filters the convective operator in order to stop the creation of smaller scales near the grid cut-off, but preserves the skew-symmetry so that good numerical stability is preserved upon regularization.

This paper generalizes the symmetry-preserving discretization and regularization models to compressible flow. Some symmetry-preserving discretizations for compressible flow have already been proposed [1, 2]. These discretizations start from the conservative form of the Navier-Stokes equations, identify the mathematical equalities needed to demonstrate conservation of kinetic energy by convective transport, and preserve these equalities at the discrete level. This procedure suffices for the derivation of highly stable symmetry-preserving discretizations, but the mathematical framework is not concise enough to facilitate the derivation of symmetry-preserving regularization models for compressible flow.

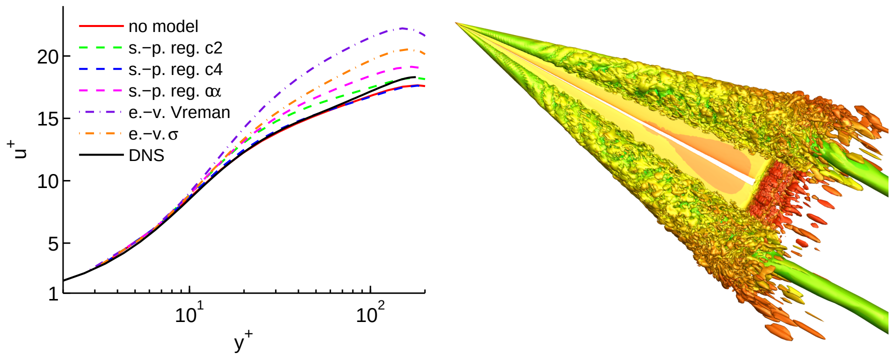

Figure 1: Simulations of compressible plane channel flow (Mach = 0.2, $\mathrm{Re}_\tau \approx 180$, $\Delta y^+_{\mathrm{min}} = 3.4$) with symmetry-preserving regularizations and eddy-viscosity large-eddy simulation models (left) and isosurfaces of the instantaneous axial vorticity in a delta wing simulation without artficial dissipation (right).

In this paper a new explanation of conservation in compressible flow is proposed. Instead of starting from the conservative form of the compressible Navier-Stokes equations, the compressible Navier-Stokes equations are expressed as evolution equations for the variables $\sqrt{\rho}$, $\sqrt{\rho}u_i/\sqrt{2}$ and $\sqrt{\rho e}$. In these variables the convective operator in each evolution equation is $c(\vec{u})\phi = \frac{1}{2}\vec{u}\cdot\nabla\phi + \frac{1}{2}\nabla\cdot(\vec{u}\phi)$ and the conservation of mass, momentum, kinetic energy and internal energy can all be explained from the skew-symmetry of the operator $c(\vec{u})$. This new explanation facilitates an elegant mathematical analysis of the existing symmetry-preserving discretizations. Moreover, the convective operator $c(\vec{u})$ is equivalent to the skew-symmetric form of the incompressible convective terms, which allows straightforward derivation of symmetry-preserving regularizations for compressible flow.

The excellent long-time stability of symmetry-preserving discretization for compressible flow is demonstrated in a simulation of subsonic plane channel flow and a simulation of the flow over a delta wing (see figure 1). The symmetry-preserving discretization for compressible flow is found to be a good implicit large-eddy simulation model for channel flow (see figure 1 left). The symmetry-preserving regularizations compare favourable to advanced eddy-viscosity models in simulations of channel flow at $\mathrm{Re}_\tau \approx 180$.

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