

# A MODAL ANALYSIS METHOD FOR STRUCTURAL MODELS WITH NON-MODAL DAMPING

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A general method for the modal decomposition of the equations of motion of damped multi-degree-of-freedom-systems is presented. Two variants of the method are presented, both based on the corresponding eigenvalue problem of the damped structure with symmetric but non-modal damping matrix. The first variant operates with the complex right eigenvectors, normalized relative to the general mass matrix. The second presented variant includes the complex left and right eigenvectors, orthonormal relative to the general stiffness matrix. After initial partitioning of the equations of motion a real modal transformation matrix is built by a combination of two complex transformations, developed analytical by the aid of computer algebra software. For the general case of damped structures with non-diagonalisable symmetric damping matrix a modal analysis can be performed in real arithmetic. Modal damping as a special case is also considered.

As far as the method is based on initial complex eigenvalue solution, a reliable eigenmode-solver for large complex eigenvalue computations must be available to deliver (in real life problems) at least a few of the lowest conjugated complex eigenmode pairs of the structural model. In an application of the first proposed method, given in [1]-[3] (analysis of a FE-model of off-shore WTG interacting with sea ground and water environment), it has been used an implicitly restarted Arnoldi/Lanczos method [6], [7] to solve the complex eigenvalue problem.

Besides the derived modal transformation in real space the suggested new approach retains the common advantages of the classic modal solution:

- 1) The equations of motion are uncoupled into SDOFS block equations, and
- 2) An uncompleted modal transformation, employing only a few ( $k$ ,  $k \ll n$ ) right and left conjugated complex eigenvector pairs to derive the suggested real basis, leads with sufficient numerical accuracy to the total time response of all  $n$  DOF after the final back coordinate transformation; hence, only a few ( $k$ ) real modal SDOFS-equations are to be numerically integrated.

Two numerical examples demonstrate the performance of the presented modal solution method. It can be applied in structural systems containing different damping and energy-loss mechanism in various parts of the structure and also in structure-environment interaction problems, where a non-modal damping matrix is occurring.

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