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A DUAL LAGRANGE METHOD WITH REGULARIZED FRICTIONAL CONTACT CONDITIONS: MODELLING MICRO SLIP*

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Solving non-linear contact problems within the FEM framework is still a challenging task from both the mathematical and the engineering point of view. In recent years the dual mortar method [1] has become of great interest for solving contact problems, since it enforces the contact conditions in variationally consistent way without increasing the algebraic system size. But most publications in this area restrict their considerations to hard contact in normal direction and a perfect Coulomb law in tangential direction [2, 3]. For use in structural dynamics it is important to model the transition from sticking to slipping in a physical correct way in order to reproduce measured frictional damping. Taking the surface roughness on the micro scale into account, there are three different states in the tangential contact on the micro scale [4]: for small shear stresses one gets a linear elastic stick region, with increasing shear stress, one reaches the region of micro slip before the whole contact zone is slipping and one reaches macro slip, see figure 1. Associated mathematical models were introduced by Oden and Pires [5].

The micro slip has significant impact on the dynamic behaviour of the body like in a bolted connection in the casing of a jet turbine. In order to model this behaviour numerically within a dual mortar framework, we start with a perturbed Lagrange formulation [6] in tangential direction and introduce an regularization Operator G^c_τ depending on the shear

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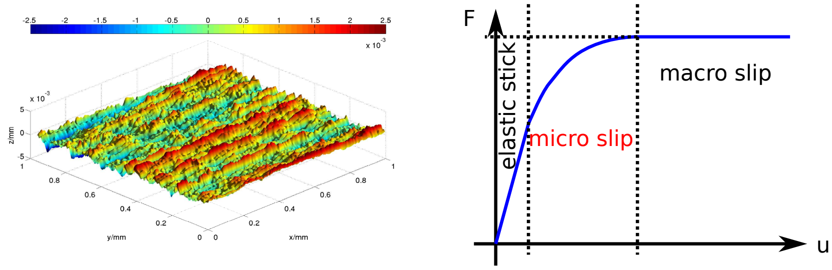

Figure 1: A milled steel surface on the micro scale and the resulting tangential contact law taking this surface roughness into account

stress λ_τ to model the elastic stick:

$$\Phi(\lambda_n, \lambda_\tau) := \|\lambda_\tau\| - \mu\lambda_n \leq 0 \quad \wedge \quad [\dot{u}]_\tau - G^c_\tau(\dot{\lambda}_\tau) = \dot{\gamma}\frac{\lambda_\tau}{\|\lambda_\tau\|} \quad \wedge \quad \dot{\gamma} \geq 0 \quad \wedge \quad \dot{\gamma}\cdot\Phi = 0$$

A mass lumping technique is used to exploit the full advantages of the duality pairing. This leads to a regularized saddle point problem, for which a non-linear complementary function and thus a semi-smooth Newton method can be derived. In order to model the micro slip the approach has to be adapted.

The derived numerical algorithms were implemented within the free FEM package CalculiX. Selected numerical examples illustrate the robustness and applicability to real life contact problems of the newly derived dual mortar formulation.

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