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A COMPARISON BETWEEN MONTE CARLO AND POLYNOMIAL CHAOS EXPANSION TECHNIQUES IN RESERVOIRS SIMULATIONS

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1 Introduction

Suppose we want to compute the first two moments of a X be a random variable. For matters of simplicity we will restrain the following explanation to the unidimensional case. The multidimensional case can be easily derived from it (see, e.g., [2]). Assume the distribution function of X is completely unknown we can always apply Monte Carlo method. In other words, let $\{x_i\}_{i\in\mathbb{N}}$ a sequence of independent random samples of X then $E(X) = \lim_n \frac{1}{n}\sum_{i=1}^{n} x_i$ On other hand, if we know X's distribution then we compute the above expression by other techniques. In fact, the expection is equivalent to solve the following integral:

$$E(X) = \int_{-\infty}^{\infty} x\text{pdf(x)}dx, \tag{1}$$

where pdf denote the density of X.

Note that the integral of Equation can be solved by any numerical method. Polynomial Chaos Expansions (PCE) were introduced by [3] in 1938 but only applied several years later by [2]. Let $f \in L^2(\mathbb{R})$ arbitrary and X a random variable with known distribution fuction. Consider $f(X)$ be the result of a black-box simulation, where $f(X)$ is a random variable with unknown distribution function. PCE technique rewrite f towards an orthonormal polynomial basis expansion $\{\gamma_i\}_{i\in\mathbb{N}}$ $f(x) = \sum_{i=1}^{\infty} \beta_i\gamma_i(x)$, where $\{\beta_i\}_{i\in\mathbb{N}}$ is a scalar set given by $\beta_i = \langle f;\gamma_i\rangle = \int f(x)\gamma_i(x)\mathrm{pdf(x)}$ for each $i = 1, 2, 3, \ldots$. It can be verified that $E[f(x)] = \beta_0$ and $\mathrm{Var}[f(x)] = \sum_{i\in\mathbb{N}} \beta_i^2$, where Var[.] denotes the variance (see, e.g., [2]). In other words to solve our original problem we just need to compute the coefficients, i.e., solve an integral. Therefore the PCE technique seems to be costless in computacional terms than Monte Carlo Method.

In this work we compare the previous techniques for 1 to 4 variables, repectively. We have used the model of PUNQ-S3 and UNISIM reservoirs to made our tests.

2 Monte Carlo × PCE in Resevoir Simulations

For the unidimensional case numerical tests have showed PCE technique need only five simulations in the IMEX to compute the expectation and variance with a good precision. Instead, the Monte Carlo technique need at least 30.000 simulations to get similar results.

PCE		
Expectation	Variance	Simulations
2,07946E+11	1,29E+20	5
Monte Carlo		
Expectation	Variance	Simulations
2,04327E+11	1,22E+20	30000

Similar results were obtain for the case with 3 variables running a few more simulations. On other hand an increase in the number of random variables exponentially increases the number of simulations required by PCE, problem known as “Curse of Dimensionality”. In our case, the number of simulations with 6 variables are approximately the same of Monte Carlo. To solve or at least reduce the number of simulations we are applying Sobol Sequences Sampling as suggested in [1]. The results are still on going but they seemed to be promising. As an example we compute the integral of function $f(x_1, \ldots, x_6) = \exp(x_1^3 x_4 + x_3^{x_2} x_5)\sin(x_1 - x_5 x_6)$. Applying Sobol with 4.096 points: $0,4103$. On other hand, we just got the same result with Monte Carlo (20 rounds) after 2.000.000 simulations.

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