11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

WEAKENED WEAK (W2) FORM METHODS: THEORY, FORMULATION AND APPLICATIONS

Liu GR[1,2]

[1] Department of Aerospace Engineering and Engineering Mechanics, University of Cincinnati, USA
[2] Consultant, College of Mathematics, Taiyuan University of Technology, China

Key Words: *weakened weak formulation, smoothed point interpolation methods (S-PIMs), smoothed FEMs (S-FEMs), generalized Galerkin methods, numerical methods.*

To overcome shortcomings of the standard finite element method (FEM), a class of numerical methods, known as the weakened weak (W2) form methods, have been recently developed [1]. The W2 methods have theoretical foundation on the so-called G space theory [2] that ensures the stability and convergence of the W2 methods. The G space theory was established based on the generalized gradient smoothing method (GSM) [3] developed based on the strain smoothing techniques [4]. The generalized GSM uses functions that may be discontinuous created using the general point interpolation methods (PIM) [5] using essentially only nodes in the problems domain. The W2 methods may use a background cells for efficient node selection and creation of smoothing domains. The background cells can be usually the simplest T-mesh (Triangular for 2D and Tetrahedral for 3D), and any FE mesh is usable. Therefore, W2 methods can also be, in general, viewed as methods that combine the meshfree methods [5] with FEM [6]. Because the W2 methods are often found working well with the T-mesh, and possess advantages of both meshfree and FEM methods, they are very powerful.

Because of the G space theory is much more general than the counterpart of Hilbert space theory, much more diversified W2 methods can be developed [5]. So far, the developed W2 methods include the smoothed PIM (S-PIMs) [1] that use the PIM shape functions. The S-PIM can have various models depending on the use of different smoothing domains. When node-based smoothing domains are used, we have the node-based smoothed PIM or NS-PIM; when edge-based, we have the ES-PIM; when cell-based, we have the CS-PIM; and when face-based, we have the FS-PIM. One can even use combined node- and cell-based smoothing domains, leading to the so-called αS-PIM (or αPIM). Applications have showed crucial significances for these S-PIMs: (1) S-PIM models are always “softer” than the standard FEM, promising more effective numerical solutions; (2) S-PIM models give more freedom in constructing shape functions for special purposes or enrichments; (3) S-PIM models allow the use of distorted background meshes and even general n-sided polygonal meshes; (4) NS-PIM offers a much simpler tool to estimate upper bounds for many types of problems; (5) the αS-PIM can offer solutions of ultra-high accuracy.

Because S-PIMs are rooted from the meshfree methods, it is more involved in terms of formulation and model creation. A “watered-done” version of S-PIM is then proposed using the FEM shape functions, leading to the smoothed FEM (S-FEM) [8]. The S-FEM is the dimpliest version of S-PIM where the linear PIM shape functions are used. Because S-FEM uses FEM mesh, it has many similarities as FEM, and it still possesses most excellent properties of S-PIM, it is getting popularity. S-FEM models use the FE mesh but evaluate the weak form based on smoothing domains created based on cells/elements, or nodes, or edges or faces. The S-FEM also have variety of models including the CS-FEM [8][9], NS-FEM [10][11], ES-FEM)

[12], FS-FEM [13], and αS-FEM (or αFEM) [14]. The Because W2 models do not perform differentiation to the shape functions; it is extremely useful for field enrichments (e.g, at the crack-tip [15]-[20]). The S-FEM has also applied to acoustics [21] and heat transfer problems [22]. This talk with provide a general overview of the existing W2 methods including the S-PIMs and S-FEMs.

REFERENCES

[1] Liu GR and Zhang GY, *Smoothed Point Interpolation Methods -- G Space Theory and Weakened Weak Forms*, World Scientific, 2013.

[2] Liu GR, On G space theory, *Int. J. Comput. Methods*, 6(2), 257-289, 2009.

[3] Liu GR. A generalized Gradient smoothing technique and the smoothed bilinear form for Galerkin formulation of a wide class of computational methods. *Int. J. Comput. Methods*, 2008; 5(2): 199-236.

[4] J.S. Chen, C.T. Wu, S. Yoon, Y. You. A stabilized conforming nodal integration for Galerkin mesh-free methods. Int. J. Numer. Methods Eng., 2001; 50:435–466.

[5] Liu GR, *Meshfree Methods–Moving beyond the finite element method*, 2nd Edn, CRC, 2009.

[6] Liu GR and Quek, *The Finite element method – a practical course*, Elsevier (BH), 2nd Ed. 2013.

[7] Liu GR, Nguyen Thoi Trung. Smoothed Finite Element Methods. CRC, NewYork, 2010.

[8] Liu GR, K.Y. Dai, T. Nguyen-Thoi. A smoothed finite element for mechanics problems. *Computational Mechanics* 2007; 39:859–877.

[9] Liu GR, T. Nguyen-Thoi, K.Y. Dai, K.Y. Lam. Theoretical aspects of the smoothed finite element method (SFEM). *Int. J. Numer. Methods Eng*., 2007; 71: 902-930.

[10] Liu GR, T. Nguyen-Thoi, H. Nguyen-Xuan, K.Y. Lam. A node based smoothed finite element method (NS-FEM) for upper bound solution to solid mechanics problems. *Comput. Struct*. 2009;87:14-26.

[11] T. Nguyen-Thoi, Liu GR, etc. Additional properties of the node-based smoothed finite element method (NS-FEM) for solid mechanics problems. *Int. J. Comput. Methods*, 2009; 6: 633-666.

[12] Liu GR, T. Nguyen-Thoi, K.Y. Lam. An edge-based smoothed finite element method (ES-FEM) for static, free and forced vibration analyses of solids. *J. Sound Vib.*, 2009; 320: 1100-1130.

[13] T. Nguyen-Thoi, Liu GR, K.Y. Lam, G.Y. Zhang. A Face-based Smoothed Finite Element Method (FS-FEM) for 3D linear and nonlinear solid mechanics problems using 4-node tetrahedral elements. Int. J. Numer. Methods Eng. 2009; 78: 324-353.

[14] Liu GR, T. Nguyen-Thoi, K.Y. Lam. A novel Alpha Finite Element Method (αFEM) for exact solution to mechanics problems using triangular and tetrahedral elements. Comput. Meth. Appl. Mech. Eng., 2008; 197: 3883-3897.

[15] Liu GR; Jiang Y.; Chen L.; et al, A singular cell-based smoothed radial point interpolation method for fracture problems , Comput. Struct., 89(13-14), 1378-1396, 2011.

[16] Liu GR, Nourbakhshnia N, Zhang YW, A novel singular ES-FEM method for simulating singular stress fields near the crack tips for linear fracture problems, *Eng. Fract. Mech.*, 78(6), 863, 2011.

[17] L. Chen, Liu GR, Zeng K., A novel variable power singular element in G space with strain smoothing for bi-material fracture analyses, Eng.Anal.Boundary Elem.,35(12),1303-1317, 2011.

[18] Liu GR, A G space theory and a weakened weak (W2) form for a unified formulation of compatible and incompatible methods: Part I theory and Part II applications, *Int. J. Numer. Methods Eng.*, 81(9), 1093-1156, 2010.

[19] Liu GR, Zhang, GY A normed g space and weakened weak (W2) formulation of a cell-based smoothed point interpolation method, *Int. J. Comput. Methods,* 6(1), 147-179, 2009

[20] He, Z. C., Liu, G. R., Zhong, Z. H., Wu, S. C., Zhang, G. Y., & Cheng, A. G. (2009). An edge-based smoothed finite element method (ES-FEM) for analyzing three-dimensional acoustic problems. *Computer Methods in Applied Mechanics and Engineering*, 199(1-4), 20-33.

[21] Wu, SC, Liu GR, Zhang, H. O., & Zhang, G. Y. (2008). A node-based smoothed point interpolation method (NS-PIM) for three-dimensional thermoelastic problems. *Numerical Heat Transfer Part A-Applications*, 54(12), 1121-1147.