

## MODAL-BASED GOAL-ORIENTED ERROR ASSESSMENT AND ADAPTIVITY FOR STRUCTURAL DYNAMICS.

F. Verdugo<sup>1</sup>, N. Parés<sup>2</sup> and P. Díez<sup>3</sup>

<sup>1</sup>Lehrstuhl für Numerische Mechanik (LNM),  
Technische Universität München (TUM),  
Boltzmannstr. 15, D-85747 Garching, Germany.  
Email: verdugo@lnm.mw.tum.de

<sup>2</sup>Laboratori de Càlcul Numèric (LaCàN),  
Escola Universitària d'Enginyeria Tècnica Industrial de Barcelona (EUETIB),  
C/ Compte d'Urgell, 187, E-08036, Barcelona, Spain.  
Email: nuria.pares@upc.edu

<sup>3</sup>Laboratori de Càlcul Numèric (LaCàN),  
Universitat Politècnica de Catalunya (UPC),  
Jordi Girona, 13, E-08034, Barcelona, Spain.  
Email: pedro.diez@upc.edu

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The present work introduces an efficient methodology to solve the adjoint problem associated with goal-oriented error assessment in structural dynamics. This is a new approach with respect to previous goal-oriented estimates for elastodynamics [1], which use direct time integration schemes (e.g., Newmark-like methods) to solve the adjoint problem. The modal-based adjoint approximation results in an efficient goal-oriented error estimate which is applied here in two different contexts. First, a goal-oriented space-time adaptive strategy using the modal-based error estimate is proposed [2]. Second, the modal-based approach is used to assess a new type of quantities of interest referred as timeline-dependent quantities of interest [3].

The proposed modal-based strategy is particularly efficient for some particular quantities of interest. This is because the adjoint solution is stored for few vibration modes instead of for all time steps. On the other hand, the use of post-processing techniques in the space domain (flux recovery or equilibration) can be readily applied to the (spatial) description of the modes with no need of carrying out the post-processing at each time step. Moreover, the time description of the adjoint solution is known analytically once the vibration frequencies and modes are available. This simplifies the algorithmic complexity in the context of space-time adaptivity and also in the context of assessing the

novel timeline-dependent quantities of interest. The modal-based error assessment can be easily extended to other *linear* time-dependent problems.

The quality of the error estimate depends on the selected number of vibration modes to approximate the adjoint solution. The numerical results show that particular types of quantities of interest can be assessed using few vibration modes. These particular quantities account for several practical cases. Moreover, the numerical examples show that the proposed space-time adaptive strategy furnishes adapted discretizations providing more accurate results than uniform remeshing for the same number of space-time elements. On the other hand, the proposed error estimate for the novel timeline-dependent quantities of interest furnishes accurate error approximations.

## REFERENCES

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