11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20 - 25, 2014, Barcelona, Spain

Advances of Continuous-discontinuous Numerical Method Based on Lagrange Equation

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Key Words: *Lagrange equation; Discontinuous model; Representative volume element; Nodal equation.*

Almost all the physical problems can be expressed by Lagrange equation. In numerical methods, all the calculation formulas for dynamic problems could be obtained based on Lagrange equation. Furthermore, discrete rigid bodies, particle systems, continuous-discontinuous solid media and fluid media could be integrated in a unified computational framework. Relationship between generalized coordinates in Lagrange equation and unknown variables in different grids or different media in numerical method is studied. Basic elemental dynamic computational equations for different media are described. In order to quantify the fracture in representative volume element (RVE), strain strength distribution model and friction effect are introduced. Dilation, fluidization and jam effects after solid fragmentation are also studied. For quantitative expressing the catastrophic state of solid media, fracture degree is adopted. Finally, theoretical formulas for composite system constituted by particles, solids and fluids are introdued, and basic numerical cases are given.

Part I: Lagrange equation

Lagrange equations are shown in Eq. 1-2. Where, u_i , v_i are generalized coordinates, L means energy in Lagrange system, Q_i is non-conservative forces, and A represents work done by non-conservative force or internal dissipative energy of system.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial v_i}\right)+\frac{\partial L}{\partial u_i}=Q_i \tag{1}$$

$$Q_i=\frac{\partial A}{\partial u_i} \tag{2}$$

For the Lagrange system containing particles, rigid bodies, elastic solids and fluids, the energy functional can be expressed as $L=L^p+L^r+L^s+L^f+L^{\text{inter}}$. Where L^p , L^r , L^s , L^f and L^{inter} are energies related to particles, rigid bodies, solids, fluids, and interactive effects respectively.

Based on Lagrange equation, different coordinate systems, media, and element types could use different generalized coordinates (independent variable), but with the same calculation scheme form. (Fig. 1)

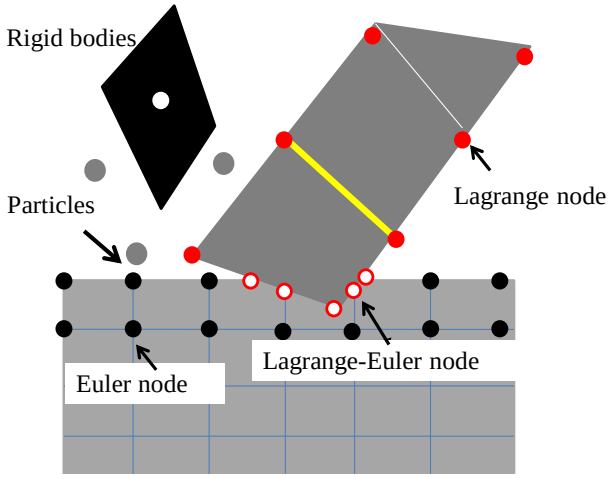

Fig. 1 Unified computational framework based on Lagrange equation

Part II: Strain strength distribution model

The basic assumption is that strain strength of RVE complies with a certain distribution law. The potential fracture plane of RVE consists of elastic microplanes and fractured microplanes. Elastic microplanes are intact parts, and fractured microplanes are the rest parts of the potential fracture plane whose strain has ever exceeded their strain strength. Nonlinear behavior of material such as yielding and strain softening can be described by this model. The typical constitutive euquatons for RVE are shown in Eq. 3-4.

$$\sigma_n = I_e\left(2G\varepsilon_n + \lambda e\right) + \begin{cases} D_b\left(2G\varepsilon_n + \lambda e\right) & \left(\varepsilon_{line-L} \le \varepsilon \le \varepsilon_{break}, \varepsilon < 0\right) \\ 0 & \left(\varepsilon_{line-L} \le \varepsilon \le \varepsilon_{break}, \varepsilon \ge 0\right) \end{cases} \tag{3}$$

$$\tau = I_e G\gamma + \begin{cases} D_b G\gamma & \begin{pmatrix} G\gamma < \left(2G\varepsilon_n + \lambda e\right)\tan\varphi \\ \gamma_{liner\text{-}L} \le \gamma \le \gamma_{break}, \varepsilon < 0 \end{pmatrix} \\ D_b\left(2G\varepsilon_n + \lambda e\right) tg\varphi & \begin{pmatrix} G\gamma \ge \left(2G\varepsilon_n + \lambda e\right)\tan\varphi, \\ \gamma_{liner\text{-}L} \le \gamma \le \gamma_{break}, \quad \varepsilon < 0 \end{pmatrix} \\ 0 & \left(\varepsilon \ge 0\right) \end{cases} \tag{4}$$

Part III: Fracture degree

Fracture degree D_f is an independent variable obtained by numerical method (Eq. 5). It is used to evaluate the state of a geological system. Where, A_c is the failure area in current state and and A_d is the failure area in disaster state.

$$D_f = A_c / A_d \tag{5}$$

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