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APPLICATIONS OF SMOOTHNESS-INCREASING ACCURACY-CONSERVING (SIAC) FILTERING FOR THE DISCONTINUOUS GALERKIN APPROXIMATION TO NONLINEAR HYPERBOLIC EQUATIONS

Jennifer K. Ryan[1] and Xiaozhou Li[2]

[1] School of Mathematics, University of East Anglia, Jennifer.Ryan@uea.ac.uk, www.uea.ac.uk/~ayz12hsu
[2] Delft Institute for Applied Mathematics, Delft University of Technology, X.Li-2@tudelft.nl

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Smoothness-Increasing Accuracy-Conserving (SIAC) filtering was originally developed for linear equations with periodic boundary conditions. Further, the original filter was designed for translation invariant meshes. Under these ideal assumptions, the filtering rids the approximation of oscillations in the error, as shown in Figure 1, while increasing the order of accuracy of the DG approximation from $k+1$ to $2k+1$ (here k is the highest degree polynomial used in the approximation). The SIAC filtered DG solution is obtained by convolving the DG approximation with a convolution kernel once - at the final time. The SIAC filtering kernel consists of a linear combination of B-splines whose weights are suitably chosen [3]. However, these are severe limitations for practical applications.

In this talk we discuss the barriers to extending this technique to nonlinear hyperbolic equations. That is, in order to make SIAC filtering applicable to nonlinear hyperbolic equations, we must address computationally efficient boundary filtering (for filtering near boundaries and shocks), nonuniform meshes as well as the theoretical extension. Previously it was demonstrated computationally and theoretically that it is possible to obtain $2k+m$, where m depends upon the flux for the DG solution [1], provided the solution is smooth. This talk incorporates a suitable modification of the filtering kernel near the boundary or shock regions, and demonstrates that we are able to conserve, and in most cases improve, accuracy and smoothness outside shock regions[2].

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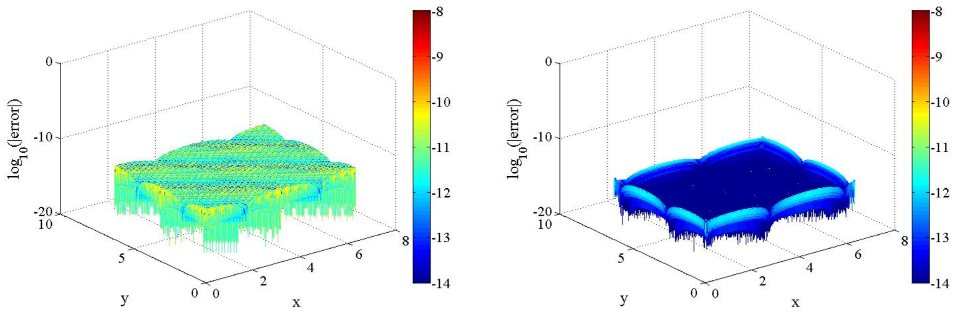

(a) DG Error (b) Post-processed

Figure 1: Comparison of the point-wise errors in log scale of the (a) DG solution and (b) the SIAC filtered solution using basis polynomials of degree $k = 4$, mesh 80×80.

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