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HIGH-ORDER ASYMPTOTIC-PRESERVING SCHEME FOR SOLVING BOLTZMANN-BGK MODEL EQUATION

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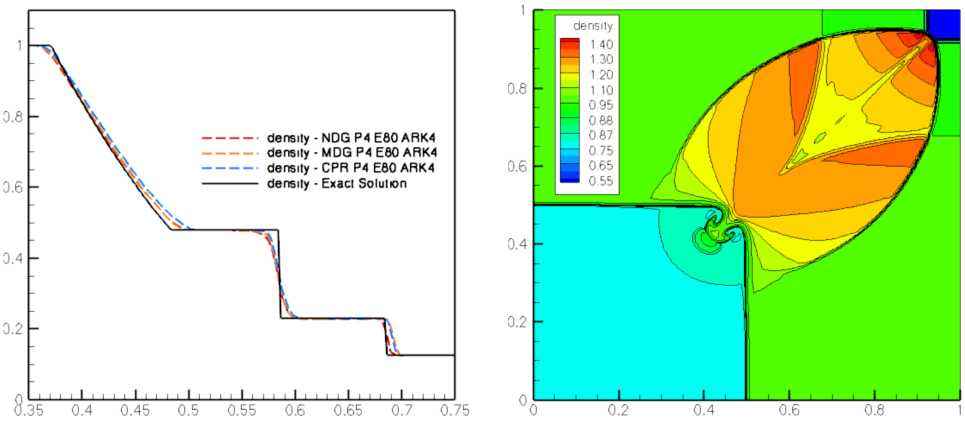
The present work is aim at developing a class of robust and economic Asymptotic-Preserving [1, 2] Boltzmann-BGK equation solver (AP-BBGK). Like many of the Botlzmann solvers, we must address discretization problem in two domains: physical space and velocity space. The present scheme, based on the work of [3], has the advantage of low memory storage when solving Boltzmann equation in one and two space dimensions. This scheme is upgraded to become Asymptotic-Preserving following ideas in [1, 2] and implemented with an additive Runge Kutta strategy [4]. By enforcing the conservation principle (Miuessens 2000) of Boltzmann collision operator we minimize the number of discrete velocities in phase-space. Finally we choose to implement high-order discontinuous schemes to explode their flexibility in discretizing the physical domain. However each scheme presents unique advantages when working under our new method. Here, we choose to compare the three most popular schemes in literature: Modal Discontinuous Galerkin (MDG) [5], Nodal DG (NDG) [6] and Correction Procedure via Resconstruction (CPR) [7, 8] by adopting the Artificial Viscosity strategy proposed in [9] and ensure a fair comparison. The local formulation of each method is implemented, and the present results are compared to that of high-order WENO [10]. Preliminary results show that the AP scheme improves the stability regions of both DG-like methods and FD-WENO scheme; in the later it allows us to evolve information with $CFL = 1.0$. Preliminary conclusions indicate that hp-adaptation must be implemented for higher-dimensional cases to achieve efficient computations. Comparisons between high order WENO schemes and DG methods for Boltzmann-BGK equation using gas dynamic problems in 2D will be presented.



(a) 1D Riemann Problem (b) 2D Riemann Problem

Figure 1: (a) Sod's shock tube solution with AP-BBGK scheme using NDG, MDG, CPR, with 80 elements, 4th-order polynomials and ARK4. Artificial viscosity is used to control spurious oscillations. (b) Solution of Configuration 12 in *Lax and Liu (1998)*, using AP-BBGK with WENO7 for a Kn= $\frac{1}{100{,}000}$ and $CFL = 1.0$ in a 200x200 mesh.

REFERENCES

[1] S. Jin. Efficient Asymtotic-preserving (AP) schemes for some multiscale kinetic equations, *SIAM J. Sci. Comp.*, Vol. **21**, 441–454, 1999.

[2] S. Pieraccini, G. Puppo. Implicit-explicit schemes for BGK kinetic equations. *J. Sci. Comp.*, Vol. **32**, Issue 1, 1–28, 2007.

[3] J.Y. Yang, J.C. Huang. Rarefied flow computations using nonlinear model Boltzmann equations. *J. Comp. Phys.*, Vol. **120**, 323–339, 1995.

[4] A. Kanevsky, M.H. Carpenter, D. Gottlieb, J.S. Hesthaven. Application of implicit-explicit order Runge-kutta methods to discontinuous-Galerkin schemes. *J. Comp. Phys*, Vol. **225**, 1753–1781, 2007.

[5] B. Cockburn. Discontinuous Galerkin Methods. *Z. angew. Math. Mech.*, Vol. **83**, 731–754, 2003.

[6] J.S. Hestaven, T. Warburton. *Nodal Discontinuous Galerkin Methods: Algorithms, Analysis and Applications*. First Edition, Springer, Text in Applied Mathematics, Vol. **54**, 2008.

[7] H.T. Huynh, High-order methods including discontinuous Galerkin by reconstructions on triangular meshes. *AAIA paper 2011-44*

[8] Z.J. Wang, H.Y. Gao. A unifying lifting collocation penalty formulation for the Euler equations. *AAIA Paper 2009-3784*

[9] P-O. Persson, J. Peraire. Sub-cell shock capturing for discontinuous Galerkin methods, *J. of Comp. Phys.*, Vol **226**, 879–896, 2007.

[10] C.-W. Shu. High order weighted essentially non-oscillatory schemes for convection dominated problems. *SIAM Review*, Vol. **51**, Issue 1, 82–126, 2009.