HIGH-ORDER ASYMPTOTIC-PRESERVING SCHEME FOR SOLVING BOLTZMANN-BGK MODEL EQUATION

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The present work is aim at developing a class of robust and economic Asymptotic-Preserving [1, 2] Boltzmann-BGK equation solver (AP-BBGK). Like many of the Boltzmann solvers, we must address discretization problem in two domains: physical space and velocity space. The present scheme, based on the work of [3], has the advantage of low memory storage when solving Boltzmann equation in one and two space dimensions. This scheme is upgraded to become Asymptotic-Preserving following ideas in [1, 2] and implemented with an additive Runge Kutta strategy [4]. By enforcing the conservation principle (Minesens 2000) of Boltzmann collision operator we minimize the number of discrete velocities in phase-space. Finally we choose to implement high-order discontinuous schemes to explode their flexibility in discretizing the physical domain. However each scheme presents unique advantages when working under our new method. Here, we choose to compare the three most popular schemes in literature: Modal Discontinuous Galerkin (MDG) [5], Nodal DG (NDG) [6] and Correction Procedure via Reconstruction (CPR) [7, 8] by adopting the Artificial Viscosity strategy proposed in [9] and ensure a fair comparison. The local formulation of each method is implemented, and the present results are compared to that of high-order WENO [10]. Preliminary results show that the AP scheme improves the stability regions of both DG-like methods and FD-WENO scheme; in the later it allows us to evolve information with $CFL = 1.0$. Preliminary conclusions indicate that $hp$-adaptation must be implemented for higher-dimensional cases to achieve efficient computations. Comparisons between high order WENO schemes and DG methods for Boltzmann-BGK equation using gas dynamic problems in 2D will be presented.
Figure 1: (a) Sod’s shock tube solution with AP-BBGK scheme using NDG, MDG, CPR, with 80 elements, 4th-order polynomials and ARK4. Artificial viscosity is used to control spurious oscillations. (b) Solution of Configuration 12 in Lax and Liu (1998), using AP-BBGK with WENO7 for a Kn= \( \frac{1}{100,000} \) and CFL = 1.0 in a 200x200 mesh.

REFERENCES


