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HIGH-ORDER HYBRID METHODS FOR ELASTIC WAVES

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In simple settings it is easy to see that high-order methods are most efficient for propagating waves over many wavelengths. This has prompted researchers to develop and implement spectral element methods to simulate elastic waves, particularly for seismic applications, e.g. [1, 2]. Advantages of the unstructured grid methods considered in these works include the possibility to easily model complex geometric structures as well as guaranteed stability based on energy estimates. A downside, however, is a computational cost which grows nonlinearly with method order. The root of this high cost is the artificial stiffness associated with high-degree polynomial differentiation operators defined on an entire element [3], possibly combined with local dense matrix operations. As a result, method orders used in practice are often limited to $4-5$ in space.

High-order structured grid methods, on the other hand, only entail work which grows linearly with method order. The stiffness associated with differentiating polynomials throughout their domain of definition is avoided and dense linear algebra never arises. Thus the full efficiency of high-order methods in terms of minimizing the number of points-per-wavelength can be achieved. However, multiblock mapped grids must be used, which limits the geometric complexity which can be treated. In addition, robustly stable methods satisfying energy estimates based on the so-called summation-by-parts (SBP) property require order reduction at boundaries [4, 5], and extensions to complex boundary conditions encountered in seismology have generally been limited to second order [6].

The approach we examine in this work is aimed at leveraging the advantages of both classes of spatial discretization schemes by utilizing hybrid structured-unstructured grids: an unstructured portion near complex geometric features and structured portion in the remainder of the computational domain [7, 8]. Here our method on the unstructured portion of the grid will be a nodal discontinuous Galerkin method as in [2]. On the structured portion we will demonstrate applications using two novel structured grid methods: Hermite spectral elements [9] and variational difference methods [10]. These are alternatives to the SBP methods used in [8] which are also energy-stable.

Basic applications of the method will include examples with both free-surface boundary conditions as well as high-order radiation boundary conditions. The latter are based on our complete radiation boundary condition (CRBC) construction, which we have demonstrated to be stable and accurate for the elastic wave system. [11, 12].

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