DISCONTINUOUS PETROV–GALERKIN METHOD WITH OPTIMAL TEST FUNCTIONS. PROGRESS REPORT

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Past June marked the 4th anniversary of the first two ICES Reports \cite{1} in which Jay Gopalakrishnan and I presented a new FE method that automatically guarantees discrete stability by means of a Petrov-Galerkin scheme with optimal test functions computed on a fly. The main idea is very simple: compute (approximately) and use test functions that realize the supremum in the inf-sup condition - the best test functions you can have. Surprise or not, we arrive at a minimum residual method (generalized least squares) in which the approximate solution delivers the best approximation error in a special “energy” (residual) norm.

20+ papers and three Ph.D. dissertations later (see \cite{2} a recent review, and workshop presentations at \url{https://sites.google.com/site/workshoplmr}), we have managed to establish a fairly complete theory and understanding of the methodology, its potentials and limitations. The presentation will consist of three parts. I will begin with a short review of the main technology focusing on the application aspects: ultraweak variational formulation, computation of element matrices and evaluation of residual. I will flash then a number of representative examples for wave propagation, compressible and incompressible Navier-Stokes from Jamie’s, Jesse’s and Nate’s dissertations. We shall focus on issues of robustness (uniform stability in perturbation parameter) and stability in the coarse mesh regime. I will finish with a short discussion of the technology in context of polygonal elements of arbitrary shape.

REFERENCES
