11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
July 20–25, 2014, Barcelona, Spain

MULTILEVEL ESTIMATION OF RARE EVENTS

Elisabeth Ullmann[1] **and Iason Papaioannou**[2]

[1] Department of Mathematical Sciences, University of Bath, Claverton Down, Bath BA2 7AY, United Kingdom (E.Ullmann@bath.ac.uk), `http://people.bath.ac.uk/eu204/`

[2] Engineering Risk Analysis Group, TU München, Theresienstr. 90, 80333 München, Germany (iason.papaioannou@tum.de), `http://www.era.bgu.tum.de`

Key words: *Reliability, Subset simulation, Multilevel method, Markov Chain Monte Carlo*

Estimation of failure probabilities is a fundamental problem in reliability analysis and risk management of engineering systems with uncertain inputs. We focus on systems described by partial differential equations (PDEs) with random coefficients. Failure events occur if some output process exceeds a given threshold. Monte Carlo simulation methods estimate the probability of failure events by sampling the random inputs and solving the associated PDE to obtain samples of the output process. However, in modern applications with highly resolved physical models and a possibly high-dimensional sample space the cost to obtain only a single output sample is nontrivial. If a large number of samples is required then the total computational cost might become prohibitively high.

Typically, we are interested in estimating small failure probabilities P_f associated with rare events. It is well known that the standard Monte Carlo method is very inefficient for the estimation of small probabilities (the number of samples required is of the order of $1/P_f$ which is extremely large for small P_f). More sophisticated sampling methods reduce the computational cost by reducing the number of samples and/or the cost per sample.

We employ subset simulation (Au & Beck, 2001) which has been developed for the estimation of small probabilities in high-dimensional sample spaces. Subset simulation reduces the total number of samples by decomposition of the sample space into a sequence of nested, partial failure sets. However, the physical discretization of the engineering system - typically done by finite elements - is fixed in each failure set and sampling is still computing-intensive.

Multilevel Monte Carlo (MLMC) methods (Heinrich, 2001; Giles, 2008) have been applied recently for PDE-based simulations with random input data, see the pioneering works (Barth et al., 2011) and (Cliffe et al., 2011). For discretized PDEs with random coefficients MLMC estimators reduce the cost per sample by decomposition of the physical space using a hierarchy of finite element meshes.

We discuss a novel multilevel approach to subset simulation where the failure regions are computed on a hierarchy of finite element meshes. This reduces the computational workload since simulations on coarser finite element meshes are less expensive. We illustrate properties of the new method by applying it to a simple 1D displacement problem. In addition, we estimate the probability of a breakthrough event in a 2D flow cell in a random porous medium.

References

S.-K. Au and J. L. Beck, *Estimation of small failure probabilities in high dimensions by subset simulation*, Prob. Eng. Mech., 16 (2001), 263–277.

S. Heinrich, *Multilevel Monte Carlo methods*, Lecture Notes in Large Scale Scientific Computing, 2179 (2001), 58–67.

M. B. Giles, *Multilevel Monte Carlo path simulation*, Operations Research, 56 (2008), no. 3, 607–617.

A. Barth, Ch. Schwab, and N. Zollinger, *Multi-level Monte Carlo finite element method for elliptic PDE's with stochastic coefficients*, Numer. Math., 119 (2011), no. 1, 123–161.

K. A. Cliffe, M. B. Giles, R. Scheichl, and A. L. Teckentrup, *Multilevel Monte Carlo methods and applications to elliptic PDEs with random coefficients*, Comput. Visual. Sci., 14 (2011), no. 1, 3–15.