11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
E. Oñate, J. Oliver and A. Huerta (Eds)

METHOD OF MODEL REDUCTION FOR ELASTIC MULTIBODY SYSTEMS

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Key Words: *elastic multibody systems, floating frame of reference, model order reduction.*

Abstract. In this paper we propose a new method of linear model order reduction for elastic multibody systems. One of the drawbacks of traditional reduction methods is a lack of possibility to tune a reduced model for certain transfer paths and certain frequency ranges. Our approach makes it possible to overcome these problems. The method relies on the idea of line fitting of model transfer functions. We apply the method to a model of flexible bar, evaluate properties of the reduced order model in time and frequency domains, and compare the results with the results of widely used Craig-Bampton approach. The tests show that the proposed method generates reduced models with lower order, higher accuracy, and better condition number of system matrices, but it takes more time for the construction of coordinate transformation matrix. However, the computational cost of our procedure for moderate dimensional models remains low.

1 INTRODUCTION

Numerical simulation is an essential part of development process for modern technical products. It makes possible to evaluate behaviour of a system before its construction and, as a result, to optimize model's design and performance, to identify possible operation problems.

The real behaviour of a mechanical system cannot be described without taking into account deformation effects. For some applications the difference between the rigid- and elastic-body behaviour is negligible, but for others, as light-weight, high-precision and high-speed mechanisms, the difference can be significant. Currently deformation effects appear in applications of many engineering fields: robotics, biomechanics, and vehicle and aircraft dynamics. This fact motivates investigation of simulation methods for systems containing flexible bodies.

The most efficient way to describe dynamics of elastic multibody systems (EMBS) undergoing small deformations is a floating frame formulation [1]. According to this method the total motion of elastic body is divided into two parts: rigid body motion represented by the motion of body reference frame, and deformations with respect to this frame.

Dynamic formulation of elastic body leads to a set of time- and space-dependent partial differential equations (PDEs). These equations can be solved analytically only for models having simple geometry. For bodies having complex forms the set of PDEs is transformed to a set of ordinary differential equations (ODEs) by means of spatial discretization. For this purpose the finite element method (FEM) is usually used. The high precision demands and/or complicated body geometry make it necessary to use fine discretization. This increases the number of flexible coordinates and makes the simulation computationally expensive. If the deformations are small, engineers usually resort to the help of model reduction techniques. These methods allow representing the elastic motion in an efficient way and, as a result, to speed up the simulation process.

In this article we describe a new linear model order reduction approach that solves one of the principal problems of classical reduction techniques.

The structure of this contribution is as follows. In Section 2 we provide an overview of related work and identify a gap we aim to fill in. Section 3 touches upon the fundamentals of modelling of EMBS. The new model reduction method is introduced in Section 4. Further, in Section 5 we apply the method to a flexible bar and perform validation tests. Finally, the obtained results and further work on the subject are discussed in Section 6.

2 PROBLEM STATEMENT

From the user's viewpoint the following aspects of model reduction are of special interest: the fidelity and order of reduced model, possibility to emphasize a certain frequency range of interest, estimation of error introduced by the reduction process, preservation of second order structure of equations of motion, computational efficiency of the reduction method, and the possibility of using the method by non-expert users.

Over many decades, model order reduction methods were on the focus of intensive research. The most widely used techniques in structural mechanics and elastic multibody dynamics can be divided into three categories: modal truncation, condensation [2, 3] and Component Mode Synthesis (CMS) [4, 5, 6].

More recently, a few alternative reduction methods have come from the field of control theory: Singular Value Decomposition (SVD), Krylov subspace, and SVD-Krylov based approaches [7, 8, 9, 10].

The family of SVD methods can a priori estimate the error introduced by the reduction, which makes it possible to automate the reduction process. However, due to the high method complexity the approach can be applied only for small-dimensional finite element models. The fundamental characteristic of the Krylov subspace based techniques is a low computing cost. This property enables the reducing of extremely high-dimensional systems. Nowadays, the application of these techniques in EMBS is on the focus of intensive research.

Next, we consider the traditional methods of model order reduction. The modal truncation approach relies on the concept of system eigenvalue decomposition. The method transforms the original FE coordinates into the modal coordinates that reside in the subspace spanned by the set of eigenvectors. The convergence of method is usually slow, because the spatial distribution of loads is not considered. In addition, it's impossible to tune the reduced model for certain frequency ranges and the error of reduction is initially unknown.

The category of condensation techniques is composed of static and dynamic condensation methods. Nowadays the static condensation method is identified with the Guyan reduction procedure. According to this approach all degrees of freedom (DoFs) of a flexible body are partitioned into the sets of master and slave DoFs. The method is based on the assumption that there are no forces applied to the slave DoFs. The Guyan reduction gives a good approximation for a low-frequency spectrum of FE model, but when the high frequency motion is considered, the influence of inertia terms is significant, therefore the method becomes inaccurate.

The dynamic reduction method was developed for mechanical systems that undergo harmonic excitation. In this approach the equations of motion are transformed into the frequency domain using the Laplace transformation. The coordinate reduction matrix is calculated for a given excitation frequency using the algorithmic scheme of the Guyan method. The dynamic condensation method takes into account the influence of inertial terms, but the accuracy of the reduced model is limited to the spectrum defined around the chosen frequency.

Another classical approach of model reduction is the component mode synthesis (CMS). The approach comprises a number of methods, one of which is the Craig-Bampton method. The Craig-Bampton coordinate transformation matrix consists of the combination of fixed boundary normal modes and constraint modes that account for local effects at boundaries. The method provides an accurate approximation for low and medium eigenfrequencies and corresponding eigenvectors. Nowadays, the Craig-Bampton reduction procedure is implemented in many FE software packages. The main drawback of the method is that the quality of reduced model highly depends on the selection of master DoFs. Besides, the choice of dominant modes of deformation is a difficult task that requires much engineering experience and insight into a problem under consideration.

Along with the advantages, the traditional techniques of model reduction have several limitations. The main drawback is the lack of possibility to tune certain frequency ranges of the reduced model. Besides, the classical methods can generate the reduced models having undesirable for simulation properties, e.g. the erroneous high-frequency spectrum makes matrices of the reduced model ill-conditioned and negatively affects an integration time step. In addition, the order of reduced models is usually large.

In this contribution we propose a new linear model order reduction approach. The method is based on the approximation of transfer functions and gives the user possibility to tune a reduced model for certain frequency ranges and certain transfer functions. Besides, the new method generates reduced models with smaller order and better condition number than the traditional methods. In addition, the approach can be employed in the context of EMBS.

3 FUNDAMENTALS OF ELASTIC MULTIBODY DYNAMICS

3.1 Modeling of multibody dynamics

The equations of motion of a single unconstrained body can be derived using d'Alembert's or Jordain's principles. This leads to the following representation:

$$\begin{bmatrix} \mathbf{M}_r^i & \mathbf{M}_{re}^i \\ \mathbf{M}_{er}^i & \mathbf{M}_e^i \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}_r^i \\ \ddot{\mathbf{q}}_e^i \end{Bmatrix} + \begin{Bmatrix} \mathbf{0} \\ \mathbf{K}_e^i \cdot \mathbf{q}_e^i + \mathbf{D}_e^i \cdot \dot{\mathbf{q}}_e^i \end{Bmatrix} = \begin{Bmatrix} \mathbf{h}_r^i \\ \mathbf{h}_e^i \end{Bmatrix} \tag{1}$$

The superscript i refers to the number of body. The vector of coordinates consists of the rigid body coordinates $\mathbf{q}_r$ and the elastic coordinates $\mathbf{q}_e$. The vector $\mathbf{q}_r$ includes 3 translational and 3 rotational coordinates; the vector $\mathbf{q}_e$ has the length N equal to the number of degrees of freedom of the finite element model. Thus, the mass matrix is divided into the rigid body part $\mathbf{M}_r \in \mathbb{R}^{6\times6}$, the elastic part $\mathbf{M}_e \in \mathbb{R}^{N\times N}$, and coupling parts $\mathbf{M}_{re} = \mathbf{M}_{er}^T$. The matrices $\mathbf{K}_e$ and $\mathbf{D}_e$ represent the stiffness and damping matrices of the elastic body. The Coriolis, centrifugal, and the external applied forces are summarized in the force vector $\mathbf{h}$.

The equation of motion of the whole EMBS can be written as

$$\mathbf{M}(\mathbf{q}) \cdot \ddot{\mathbf{q}} + \mathbf{D} \cdot \dot{\mathbf{q}} + \mathbf{K} \cdot \mathbf{q} = \mathbf{g}(\dot{\mathbf{q}}, \mathbf{q}, t), \tag{2}$$

where $\mathbf{q}$ is a vector of generalized coordinates of the whole system, $\mathbf{g}$ includes the vector $\mathbf{h}$ and the vector of constraint forces. The theory of EMBS is described in details in [11, 12].

3.2 Modeling of elastic bodies

In this subsection we focus on the concept of model order reduction. Let **u**(t) and **y**(t) be an input distribution and an output measurement arrays respectively. The elastic part of the equation of motion (1) is transformed into a system

$$\begin{aligned} &\mathbf{M}_e \cdot \ddot{\mathbf{q}}_e(t) + \mathbf{D}_e \cdot \dot{\mathbf{q}}_e(t) + \mathbf{K}_e \cdot \mathbf{q}_e(t) = \mathbf{B}_e \cdot \mathbf{u}(t) \\ &\mathbf{y}(t) = \mathbf{C}_e \cdot \mathbf{q}_e(t), \end{aligned} \tag{3}$$

with the input matrix $\mathbf{B}_e \in \mathbb{R}^{N\times p}$ and output matrix $\mathbf{C}_e \in \mathbb{R}^{r\times N}$, where p and r are numbers of input and output coordinates respectively.

When the floating frame approach is used for modelling elastic multibody systems, the elastic degrees of freedom can be reduced by means of methods of linear model order reduction. The basic idea of model reduction is to approximate the original set of ODEs by a low-dimensional set of equations that preserves significant characteristics of the full model. The large vector of elastic coordinates $\mathbf{q}_e \in \mathbb{R}^N$ is approximated using the vector $\tilde{\mathbf{q}}_e \in \mathbb{R}^n$ of a smaller dimension $n \ll N$ and a projection matrix $\mathbf{V} \in \mathbb{R}^{N\times n}$:

$$\mathbf{q}_e(t) \approx \mathbf{V} \cdot \tilde{\mathbf{q}}_e(t). \tag{4}$$

The transformation (4) leads to the following equations of motion:

$$\begin{aligned} &\tilde{\mathbf{M}}_e \cdot \ddot{\tilde{\mathbf{q}}}_e(t) + \tilde{\mathbf{D}}_e \cdot \dot{\tilde{\mathbf{q}}}_e(t) + \tilde{\mathbf{K}}_e \cdot \tilde{\mathbf{q}}_e(t) = \tilde{\mathbf{B}}_e \cdot \mathbf{u}(t) \\ &\mathbf{y}(t) = \tilde{\mathbf{C}}_e \cdot \tilde{\mathbf{q}}_e(t), \end{aligned} \tag{5}$$

with the reduced matrices $\tilde{\mathbf{M}}_e = \mathbf{V}^T \cdot \mathbf{M}_e \cdot \mathbf{V}$, $\tilde{\mathbf{D}}_e = \mathbf{V}^T \cdot \mathbf{D}_e \cdot \mathbf{V}$, $\tilde{\mathbf{K}}_e = \mathbf{V}^T \cdot \mathbf{K}_e \cdot \mathbf{V} \in \mathbb{R}^{n\times n}$ and $\tilde{\mathbf{B}}_e = \mathbf{V}^T \cdot \mathbf{B}_e \in \mathbb{R}^{n\times p}$, $\tilde{\mathbf{C}}_e = \mathbf{C}_e \cdot \mathbf{V} \in \mathbb{R}^{r\times n}$.

4 LINE-FITTING MODEL ORDER REDUCTION APPROACH

The task of model reduction methods is to find a coordinate transformation matrix **V**. This section shows how to construct the matrix **V** according to our model reduction approach.

The use of reduced models in the context of EMBS imposes on the reduction method additional demands. First, the reduced bodies have to preserve the second order structure of equations of motion. It follows that the reduction methods based on the transformation of initial equations to the first order equations cannot be utilized. Second, the dynamics of EMBS relies on the floating frame approach, which requires that the transformation matrix **V** doesn't contain rigid body modes. Besides, some additional properties like stability and passivity are to be preserved to make the reduction applicable for EMBS.

Our reduction method approximates the dynamical behavior of elastic body by fitting transfer functions

$$\mathbf{H}(s) = \mathbf{C}_e \cdot (s^2\mathbf{M}_e + s\mathbf{D}_e + \mathbf{K}_e)^{-1} \cdot \mathbf{B}_e . \tag{6}$$

They can be determined from the system of equations (5) by using the Laplace transformation with the complex frequency $s = i\omega$. In order to obtain the projection matrix **V** without rigid body modes, we should exclude the contribution of rigid body motion form Equation (6). The translational and rotational rigid modes can be written as follows:

$$\mathbf{V}_0 = \begin{bmatrix} \mathbf{V}_{0t} & \mathbf{V}_{0r} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \vdots \\ [\mathbf{I}]^k \\ \vdots \end{bmatrix} & \begin{bmatrix} \vdots \\ [-\tilde{\mathbf{r}}]^k \\ \vdots \end{bmatrix} \end{bmatrix}, k = 1,.., N. \tag{7}$$

The matrix **I** is 3 × 3 identity matrix, the term $[\mathbf{r}]^k$ gives the undeformed position of the *k*-th node and consequently $[\tilde{\mathbf{r}}]^k$ annotates the associated skew symmetric matrix. It is assumed here that a finite element node exhibits only three translational degrees-of-freedom.

The component of transfer matrix that corresponds to rigid body motion can be found like that

$$\mathbf{H}_0(s) = \mathbf{C}_0 \cdot (s^2\mathbf{M}_0 + s\mathbf{D}_0 + \mathbf{K}_0)^{-1} \cdot \mathbf{B}_0 \tag{8}$$

where $\mathbf{M}_0 = \mathbf{V}_0^T \cdot \mathbf{M}_e \cdot \mathbf{V}_0$, $\mathbf{D}_0 = \mathbf{V}_0^T \cdot \mathbf{D}_e \cdot \mathbf{V}_0$, $\mathbf{K}_0 = \mathbf{V}_0^T \cdot \mathbf{K}_e \cdot \mathbf{V}_0 \in \mathbb{R}^{6\times 6}$ are the transformed system matrices, and $\mathbf{B}_0 = \mathbf{V}^T \cdot \mathbf{B}_e \in \mathbb{R}^{6\times p}$, $\mathbf{C}_0 = \mathbf{C}_e \cdot \mathbf{V}_0 \in \mathbb{R}^{r\times 6}$ are the reformed input and output matrices.

The elastic part of transfer matrix (6) is defined as given below:

$$\mathbf{H}_1(s) = \mathbf{H}(s) - \mathbf{H}_0(s). \tag{9}$$

Further, we are looking for a transformation matrix **V**. The idea is to approximate the transfer functions $\mathbf{H}_1(s)$ using a line-fitting approach and to express the transfer functions for non-output coordinates $\mathbf{H}_1^{\perp}(s)$ through the approximated functions of $\mathbf{H}_1(s)$. Let's consider the vector $\mathbf{x} = \begin{bmatrix} \mathbf{x}_o^T & \mathbf{x}_n^T \end{bmatrix}^T$ that defines displacements of nodes without rigid-body motion.

The vectors $\mathbf{x}_o(s) = \mathbf{H}_1(s) \cdot \mathbf{u}(s)$ and $\mathbf{x}_n(s) = \mathbf{H}_1^{\perp}(s) \cdot \mathbf{u}(s)$ define displacements of *o*utput and *n*on-output coordinates correspondently. Next, the displacement vector $\mathbf{x}$ is approximated using a coordinate transformation matrix **V** as follows:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_o \\ \mathbf{x}_n \end{bmatrix} \approx \mathbf{V} \cdot \tilde{\mathbf{x}}_o = \begin{bmatrix} \mathbf{I} \\ \mathbf{W} \end{bmatrix} \cdot \tilde{\mathbf{x}}_o \tag{10}$$

where $\tilde{\mathbf{x}}_o$ is the approximation of $\mathbf{x}_o$ and **I** is an r x r identity matrix. Substituting $\tilde{\mathbf{x}}_n(s) = \tilde{\mathbf{H}}_1^{\perp}(s) \cdot \mathbf{u}(s)$ and $\tilde{\mathbf{x}}_o(s) = \tilde{\mathbf{H}}_1(s) \cdot \mathbf{u}(s)$ in $\tilde{\mathbf{x}}_n(s) = \mathbf{W} \cdot \tilde{\mathbf{x}}_o(s)$ we obtain

$$\tilde{\mathbf{H}}_1^{\perp}(s) = \mathbf{W} \cdot \tilde{\mathbf{H}}_1(s). \tag{11}$$

We construct the matrix **W** as a trade-off between the points $s_k = i \cdot 2\pi f_k, k = 1..z$ of the frequency range of interest. The equation (11) is turned into

$$\begin{bmatrix} \mathbf{H}_1^{\perp}(s_1) & \ldots & \mathbf{H}_1^{\perp}(s_z) \end{bmatrix} = \mathbf{W} \cdot \begin{bmatrix} \mathbf{H}_1(s_1) & \ldots & \mathbf{H}_1(s_z) \end{bmatrix}, \tag{12}$$

The transfer functions for the output coordinates $\tilde{\mathbf{H}}_1(s)$ approximately fit $\mathbf{H}_1(s)$ at the frequency points of interest s_k. The decomposition of Equation (12) into real and imaginary parts yields:

$$\begin{gathered} \left[\Re\left(\begin{bmatrix} \mathbf{H}_1^{\perp}(s_1) & \ldots & \mathbf{H}_1^{\perp}(s_z) \end{bmatrix}\right) \quad \Im\left(\begin{bmatrix} \mathbf{H}_1^{\perp}(s_1) & \ldots & \mathbf{H}_1^{\perp}(s_z) \end{bmatrix}\right) \right] = \\ \mathbf{W} \cdot \left[\Re\left(\begin{bmatrix} \mathbf{H}_1(s_1) & \ldots & \mathbf{H}_1(s_z) \end{bmatrix}\right) \quad \Im\left(\begin{bmatrix} \mathbf{H}_1(s_1) & \ldots & \mathbf{H}_1(s_z) \end{bmatrix}\right) \right] \end{gathered} \tag{13}$$

that can be rewritten as

$$\mathbf{T}_o^T \cdot \mathbf{W} = \mathbf{T}_n^T. \tag{14}$$

The matrices $\mathbf{T}_o^T$ and $\mathbf{T}_n^T$ are elements of $\mathbb{R}^{2pz \times r}$ and $\mathbb{R}^{2pz \times (N-r)}$ respectively; p, r, z are the number of inputs, outputs, and reference frequency points. The matrix **W** can be found as a least-squares solution of (14). After that, the coordinate transformation matrix **V** is built using the equation (10). The order of reduced model is equal to the number of outputs r.

The set of input-output parameters for the construction of transformation matrix **V** can differ from the set of parameters defined by the user. The set can be redefined in order to take into account dynamic characteristics of full model that aren't observed on the initial transfer functions. In addition, the choice of new set of output coordinates make possible to decrease the order of reduced model without noticeable loss of reduction quality. After the projection matrix **V** is obtained, the reduced model for user defined set of coordinates is built by (5).

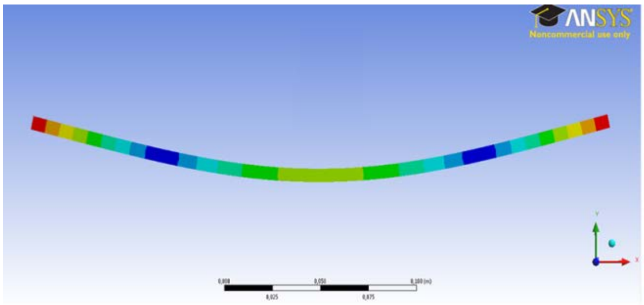
The reference frequency points have to be points of resonances and anti-resonances. Besides, additional reference frequency points can be used to enhance quality of reduced models in special ranges or points of model spectrum.

5 NUMERICAL EXAMPLE

In this section we reduce a model of flexible bar by means of our approach and the classical Craig-Bampton method, perform validation tests, and compare the results. The tests are carried out in the academic software that we developed in Maple.

5.1 Model description

The model under consideration has the following characteristics: size 6 × 8 × 300 mm, mass 0,1 kg, Young's modulus $2 \cdot 10^{10}$ Pa, damping factor beta 10^{-5}, number of degrees of freedom 1656. The modal analysis of free system gives the eigenfrequencies, illustrated in Figure 1. As a frequency range of interest we define an interval [0, 1000] Hz. The body is going to be subjected to three forces: at the middle, and the left and right ends of the model. The set of input coordinates includes DoFs of these nodes. The set of output coordinates coincides with the set of input parameters.



	Mode	Frequency [Hz]
7	7,	133,09
8	8,	171,9
9	9,	364,45
10	10,	469,8
11	11,	714,96
12	12,	918,4
13	13,	1173,5
14	14,	1501,8
15	15,	1750,7
16	16,	2158,3
17	17,	2228,
18	18,	2424,5
19	19,	2660,4
20	20,	3069,2
21	21,	3217,9
22	22,	4045,

Figure 1: Original non-zero eigenfrequencies of unconstrained body.

5.2 Initialization of model reduction methods

CMS. In order to construct a coordinate transformation matrix of the Craig-Bampton method, one has to define a set of master nodes and number of dynamic modes. The choice of the master set is crucial for the quality of the reduced order model, but it is not a trivial task. The positions of master DoFs are usually specified where forces are defined and large deformations are expected. The quantity of dynamic modes is usually chosen experimentally.

The master nodes for the Craig-Bampton procedure are presented in Figure 2. The number of constrained and dynamic modes is equal to 9 and 13 respectively, therefore the size of transformation matrix is 1656 × 22.

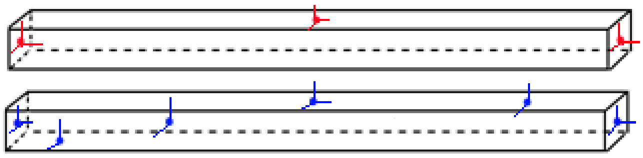


Figure 2: Interface DoFs used for computing a coordinate transformation matrix: (a) CMS method (above), (b) our method (below).

Line-fitting method. We consider the frequency range of interest, input and output coordinates as the given information for the line-fitting method. The task of engineer is to choose an appropriate order of reduced model and to allocate reference frequency points. The order of reduced model is defined by the number of output coordinates used for the construction of coordinate transformation matrix. This set can be different from the user's set of output coordinates. The order of reduced model depends on the number of transfer functions that need to be tuned and the number of eigenfrequencies in the frequency range of interest. The choice of an optimal order and allocation of output coordinates are the topics of further research.

For this test we defined 15 output coordinates, nine of them are user's outputs and six are additional DoFs. They are located as shown in Figure 2.

To specify reference frequency points we compute the transfer functions that the user intents to approximate the best. The reference points are defined at the resonance, anti-resonance and some intermediate frequencies of the interval [0,1000] Hz, see Figure 3. The total set of control points combines the reference points chosen for each transfer function. For this task we defined 20 reference points.

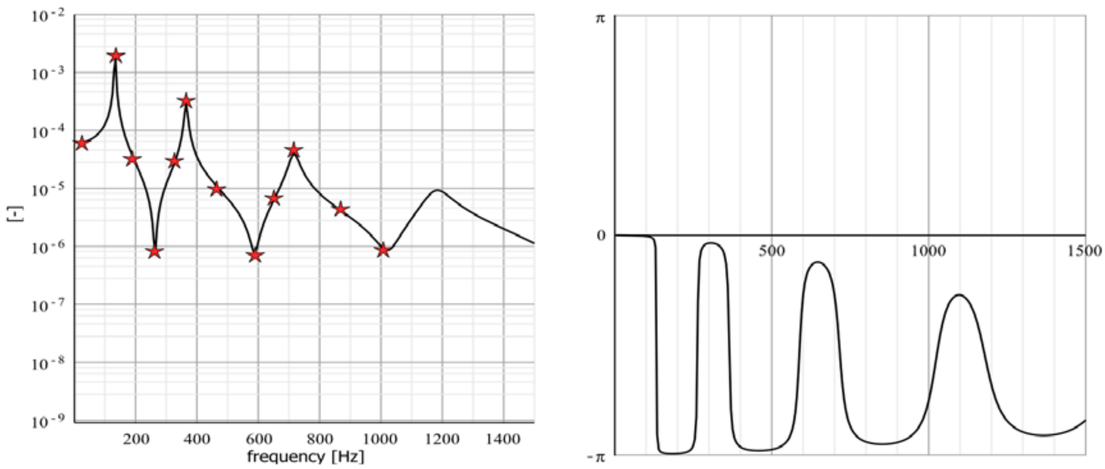


Figure 3: Choice of reference frequency points.

After the transformation matrices are generated, we perform the tests aimed at the evaluation of quality of the reduced models.

5.3 Evaluation of reduced order models

For the comparison of the reduction methods we employ two criteria: relative eigenfrequencies difference and relative error of transfer functions. Figure 4 represents the result of the former test for the eigenfrequencies in the range of interest [0, 1000] Hz, as well as for all eigenfrequencies of the reduced models.

The diagrams show that the eigenfrequencies in the frequency range of interest are accurately approximated, but the remaining eigenfrequencies are erroneous. However, the number of incorrect values for our method is small; this keeps the order of reduced model close to the minimum and eliminates the problems with an erroneous high-frequency spectrum: ill-conditioned matrices and small integration step.

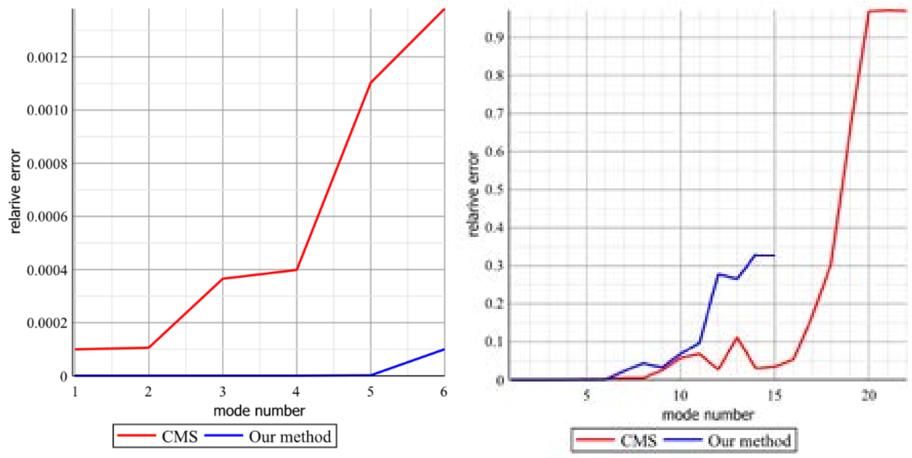


Figure 4: Relative error of model eigenfrequencies: (left) eigenfrequencies in predefined range [0,1000] Hz; (right) all eigenfrequencies of reduced models.

The quality of dynamic properties of reduced order models can be evaluated using transfer functions of original and reduced models. For MIMO systems it’s problematically to control all transfer functions separately, therefore the error of reduction is usually evaluated using the formula:

$$\varepsilon(\omega) = \frac{\left\| \mathbf{H}(i\omega) - \tilde{\mathbf{H}}(i\omega) \right\|_F}{\left\| \mathbf{H}(i\omega) \right\|_F} \tag{15}$$

where $\mathbf{H}$ and $\tilde{\mathbf{H}}$ are transfer matrices of original and reduced systems, and $\|\cdot\|_F$ denotes the Frobenius norm. The function $\varepsilon(\omega)$ shows the total error introduced by a model reduction approach in a certain frequency range.

Figure 5 represents the error of the both methods in the magnitudes of **H**.

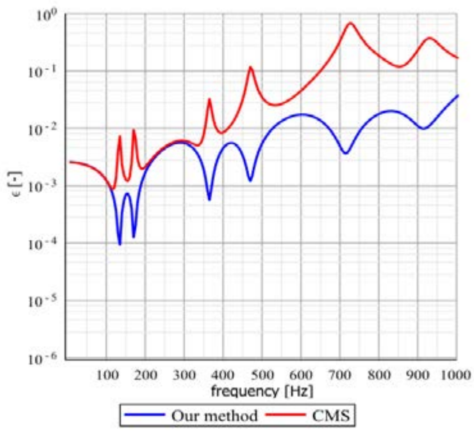


Figure 5: Relative error in magnitudes of frequency response.

The plot on Figure 5 illustrates that our method has approximated the relevant transfer functions more accurately than the Craig-Bampton approach, especially at the resonance points.

5.4 Simulation of reduced order models

The use of reduced order models in the context of elastic multibody systems implies that reduction techniques provide a meaningful approximation not only for the elastic part of (1), but also for the complete set of equations. In order to test this aspect for our method we perform simulation of elastic bar. As a floating body frame we use the Buckens coordinate system described e.g. in [11]. The equations of motion are integrated by the implicit Backward-Euler method.

Figure 6 illustrates forces applied to the flexible bar. The magnitude of end-forces is 50 N, the middle-force acts in the opposite direction and it has the value 100 N. The resultant force is equal to 0. The interest was focused on the deformation at the middle point of the model.

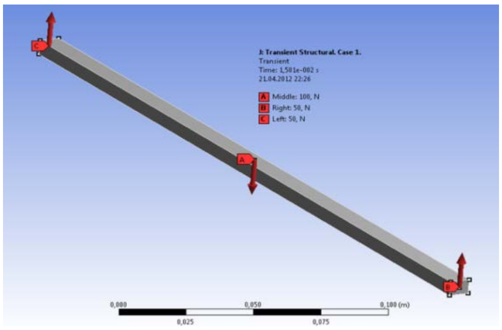


Figure 6: Forces applied to the model.

Figure 7 compares the response of full order system with the response of the system reduced by our method. The simulation of model reduced by CMS approach yields nearly the same results, but the erroneous high-frequency spectrum significantly decreases a suitable integration time step and consequently slows down the simulation.

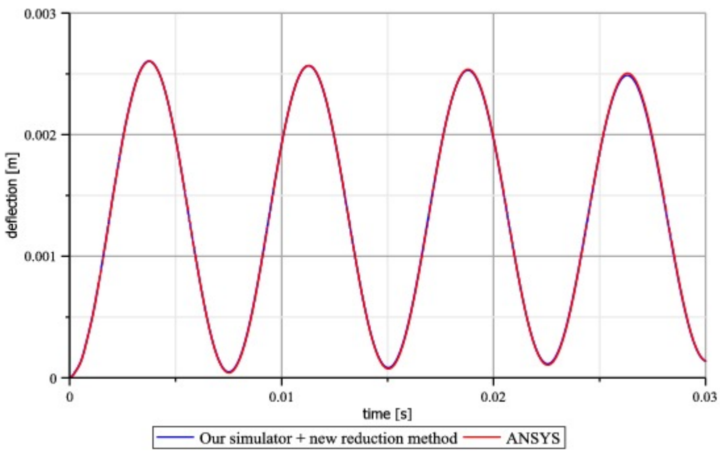


Figure 7: Transverse deflection of the bar.

Table 1 summarizes the results of analysis of the line-fitting method and compares them with the corresponding results of the classical CMS approach.

Table 1: Comparison of reduced order models

Criteria\Methods	CMS	Our method
Order	22	15
Highest eigenfrequency, (Hz)	$2 \cdot 10^5$	$4,5 \cdot 10^3$
Suitable step of integration, (s)	$2,5 \cdot 10^{-7}$	10^{-5}
Time of simulation, (hours)	$3,5$	$0,25$
Relative error of eigenfrequencies	$< 1,4 \cdot 10^{-3}$	$< 10^{-4}$
Relative error of dynamic behavior	$0,1 - 70\%$	$0,01 - 2\%$
Time of projection matrix generation, (min)	5	10

The data show that our method provides reduced order models with lower order, higher accuracy and better condition properties. The cost of coordinate transformation matrix generation is higher as compared to the Craig-Bampton approach, but it remains acceptable. In addition, the transformation matrix has to be calculated only once.

6 DISCUSSION

We found the new model order reduction method that allows overcoming the disadvantage of traditional reduction techniques: the lack of possibility to tune a reduced model for certain transfer functions and certain frequency ranges and points. The method makes possible to generate reduced models having lower order and higher accuracy than the models reduced by the classical Craig-Bampton approach. Local quality of the reduction can be improved by additional reference frequency points, whereas the quality over the whole interval is enhanced by increasing the order of reduced model. Better condition properties of reduced matrices enable to use larger step of integration and consequently to speed up a simulation process. The models reduced by our method can be used for simulation of elastic multibody systems.

The cost of transformation matrix generation is acceptable for low and moderate dimensional tasks. We have already processed models having up to 10^5 degrees of freedom.

In order to allocate the reference frequency points, it is not necessary to build the transfer functions of interest. The points of resonanses and anti-resonanses can be found solving eigenproblems of the system.

7 CONCLUSION

In this paper we presented the new method of model order reduction, applied it to the model of flexible bar, evaluated the properties of reduced order model, and compared the results with results of the traditional Craig-Bampton method based on the Component Mode Synthesis approach.

We intend to systematize the way of output coordinates selection in the process of coordinate transformation matrix construction, to tune the proposed procedure for larger finite element models, and to apply the method to elastic bodies in multibody systems with joints.

REFERENCES

[1] Wasfy, T. and Noor, A. *Computational strategies for flexible multibody systems*. Applied Mechanics Reviews, Bd. 56, Nr. 6, pp. 553-613, (2003).

[2] Guyan, J. *Reduction of stiffness and mass matrices.* AIAA Journal, Bd. 3, pp. 380-380, (1965).

[3] Leung, A. *An accurate method of dynamic condensation in structural analysis.* International Journal for Numerical Methods in Engineering, Bd. 12, Nr. 11, pp. 1705-1715, (1978).

[4] Bampton, M. C. C. and Craig, R. R. *Coupling of substructures for dynamic analyses.* AIAA, Bd. 6, pp. 1313-1319, (1968).

[5] Craig, R. *Coupling of substructures for dynamic analyses: an overview*. Proceedings of the AIAA Dynamics Specialists Conference, Atlanta, USA, (2000).

[6] Dietz, S. *Vibration and Fatigue Analysis of Vehicle Systems Using Component Modes.* VDI-Verlag, (1999).

[7] Antoulas, A. C. *Approximation of Large-Scale Dynamical Systems.* Philadelphia, PA, USA: Society for Industrial and Applied Mathematics, (2005).

[8] Lehner, M. *Modellreduktion in elastischen Mehrkoerpersystemen.* Shaker, Aachen, (2007).

[9] Fehr, J. and Eberhard, P. *Error-Controlled Model Reduction in Flexible Multibody Dynamics*. Journal of Computational and Nonlinear Dynamics, Bd. 5, Nr. 3, p. 031005, (2010).

[10] Benner, P. and Saak, J. *Efficient balancing-based MOR for large-scale second-order systems.* Mathematical and Computer Modelling of Dynamical Systems, Bd. 17, Nr. 2, pp. 123-143, (2011).

[11] Schwertassek, R. and Wallrapp, O. *Dynamik flexibler Mehrkörpersysteme: Methoden der Mechanik zum rechnergestützten Entwurf und zur Analyse mechatronischer Systeme*. Braunschweig: Vieweg, (1999).

[12] Shabana, A. A. *Dynamics of multibody systems.* Cambridge and U.K and New York: Cambridge University Press, (1998).