11th World Congress on Computational Mechanics (WCCM XI)
5th European Conference on Computational Mechanics (ECCM V)
6th European Conference on Computational Fluid Dynamics (ECFD VI)
E. Oñate, J. Oliver and A. Huerta (Eds)

SIMULATION OF THE TWO PHASE FLOW IN A WELLBORE USING TWO-FLUID MODEL

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Key words: Heat transfer, transient, tubing, pipeline, FVM, black oil.

Abstract. In a typical case, crude oil flows from reservoir up to a wellbore as liquid, however, when fluid flows inside it the pressure and temperature reduce, dissolved gas is liberated and a second phase appears. It can exist up to three phases in the fluid flow of a pipeline in a oil well: liquid (crude oil and water), gas (methane, ethane,...) and solids (sands). Ignoring sand existence (and its issues associated), the flow in the wellbore can be considered as a two-phase one. Therefore, the main objective of this work is to present a one dimensional two fluid model to simulate the two phase upward flow in the tubing of a vertical wellbore. In order to close the problem, thermo-physical properties of the fluids are evaluated using the black oil model. Furthermore, the implemented model is flow pattern dependent, and it uses the change of mass term explicitly. Firstly, the model implemented has been tested in a water faucet benchmark configuration. A good model performance has been observed.

1 INTRODUCTION

Different approaches can be used to simulate two phase flow in wellbores as a one-dimensional flow configuration. They can be grouped as follows:

1. Simplified model: it is a group of equations without a physical ground that are used to get the pressure drop.

2. Mechanistic model: it is a set of equations with a physical ground and additional closure relations to obtain the pressure drop.

3. Drift flux model: it includes a kinematic relation between liquid and gas velocity. This model can be used to get values in unsteady state.

4. Two fluid model: conservative equations (mass, momentum and energy) are defined for each phase. They involve specific terms that relate the phases to link the equations. Moreover, closure relations are needed.

Some comparisons have been published in scientific articles. Most of them compare mechanistic models with simplified models, e.g. [6, 7, 8]. These works show a better performance of some models, and it is independent of the complexity of the model.

Figure 1 shows the geometry of the well, the flow patterns that can be found in it and the variation of pressure, temperature and phase with the depth [1]. Notwithstanding two phase flow in the wellbore is common, it has been mainly studied using mechanistic or simplified models. The predictions of the models can show high discrepancies [2, 3]. Furthermore, these models are suitable only for steady state flows.

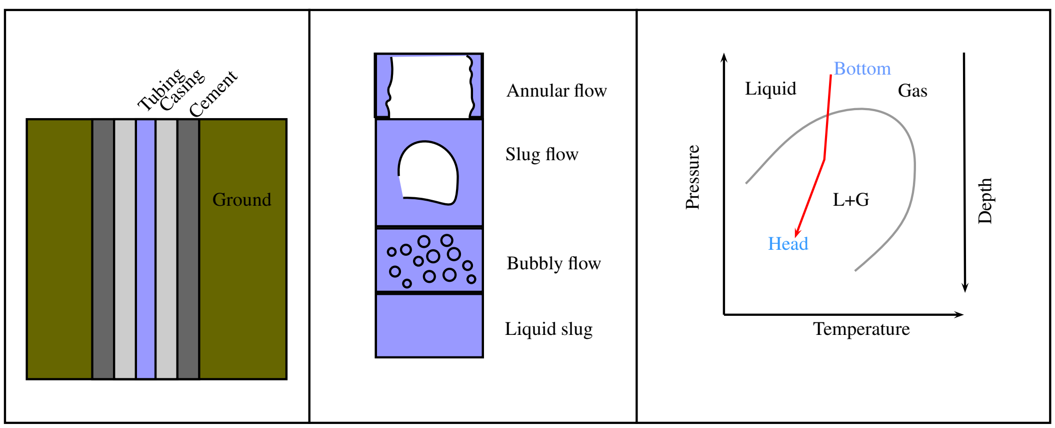


Figure 1: Basic geometry in a wellbore, flow patterns observed, and change in the flow pattern from bottom well until surface (well head).

The two fluid model has been applied in the oil industry. However, works found in the literature do not take into account the change of mass due to the liberation of gas from oil, or the model is implemented only for one flow pattern [4, 5]. Therefore, the main purpose of this work is to present a transient one dimensional two fluid model to simulate the two phase upward flow in the tubing of a vertical wellbore. In order to close the problem, thermo-physical properties of the fluids are evaluated using the black oil model. Furthermore, the implemented model is flow pattern dependent, and it uses the change of mass term explicitly.

Firstly, an explanation of the conservation equation and simplifications assumed are shown. Then, equations are reorganized to show the solution method. After that, linkage between the equations shown and properties calculated by the black oil model are explained. Finally, a first test is made with the faucet problem to compare obtained results with an analytical solution.

2 TWO FLUID MODEL

In the one dimensional two fluid model, conservation equations are applied to each phase to find an average value of the velocity (for each fluid), pressure, void fraction (and holdup) and temperature (for each fluid). Besides the number of equations, flow patterns and closure relations increase complexity in the model.

Ishii [9] presented a set of one dimensional two fluid model equations based on space and time average. This model is considered in this work, and the corresponding equations are written below.

The continuity equation is shown in (1).

$$\underbrace{\frac{\partial \langle\alpha_k\rangle \rho_k}{\partial t}}_{\text{Transient term}} + \underbrace{\frac{\partial}{\partial z}\langle\alpha_k\rangle \rho_k \langle\langle v_k\rangle\rangle}_{\text{Convective term}} = \underbrace{\langle\Gamma_k\rangle}_{\text{Change of phase term}} \qquad k = 1, 2. \tag{1}$$

In the two phase flow in wellbores, the dissolution of gas is often more important than the change of phase of some components, thereby this behaviour should be added to the equations. Dissolved gas in liquid travels at the same velocity that the liquid phase. The $\langle\Gamma_k\rangle$ term is, by definition, the amount of gas that change of phase. However, it can be modified to include the liberated gas from the liquid phase. In this case, it is a function of solubility of gas in oil R_s and water R_{sw} and not a function of enthalpy.

The momentum conservation equation can be written as follows:

$$\begin{aligned}
&\underbrace{\frac{\partial}{\partial t}\left[\langle\alpha_k\rangle \rho_k \langle\langle v_k\rangle\rangle\right]}_{\text{Transient term}} + \underbrace{\frac{\partial}{\partial z}\left[\cancelto{1}{C_{vk}}\langle\alpha_k\rangle \rho_k \langle\langle v_k\rangle\rangle^2\right]}_{\text{Convective term}} = \\
&\underbrace{-\langle\alpha_k\rangle \frac{\partial}{\partial z}\langle\langle p_k\rangle\rangle}_{\text{Pressure gradient}} + \underbrace{\cancelto{0}{\frac{\partial}{\partial z}\left[\langle\alpha_k\rangle \langle\langle \tau_{kzz} + \tau^T_{kzz}\rangle\rangle\right]}}_{\text{Shear stress in the flux direction}} \\
&-\underbrace{\frac{4\alpha_{kW}\tau_{kW}}{D}}_{\text{Wall friction}} - \underbrace{\langle\alpha_k\rangle \rho_k g_z}_{\text{Gravitational force}} + \underbrace{\langle\Gamma_k\rangle \langle\langle v_{ki}\rangle\rangle}_{\text{Change of phase momentum}} + \underbrace{\left\langle M^d_k\right\rangle}_{\text{Interfacial momentum transfer}} \\
&+ \underbrace{\cancelto{0}{\left\langle (p_{ki} - p_k)\frac{\partial \alpha_k}{\partial z}\right\rangle}}_{\text{Pressure's difference between fluid and interface}}
\end{aligned} \tag{2}$$

The following assumptions are used in order to simplify (2):

1. The convective term C_{vk} of the momentum equation uses the distribution parameter to take into account the radial profile of the velocity and flow pattern in the pipe when an averaged procedure is used. Several authors have approximated this term to 1 [4, 5, 10, 11].

2. The variation of the shear stress in the flux direction is small due to small velocity variation. This term is set to 0. Moreover, this simplification converts (2) into an unsteady convective equation with one source term.

3. Pressure difference between fluid and interface is only important for stratified flow, but, the stratified flow is not present in vertical pipes, which are studied in this work. In some cases, additional terms are used in (2) to ensure stability of the equation for horizontal flow. This is related to the Kelvin-Helmholtz Instability problem.

The energy conservation equation (in terms of enthalpy) is:

$$
\begin{aligned}
&\underbrace{\frac{\partial}{\partial t}\left[\langle\alpha_k\rangle\,\rho_k\,\langle\langle h_k\rangle\rangle\right]}_{\text{Transient term}} + \underbrace{\frac{\partial}{\partial z}\left[\cancelto{1}{C_{hk}}\,\langle\alpha_k\rangle\,\rho_k\,\langle\langle h_k\rangle\rangle\,\langle\langle v_k\rangle\rangle\right]}_{\text{Convective term}} = \\
&\quad - \underbrace{\cancelto{0}{\frac{\partial}{\partial z}\left[\langle\alpha_k\rangle\left\langle\left\langle q_k + q_k^T\right\rangle\right\rangle_z\right]}}_{\text{Diffusive term}} + \underbrace{\langle\alpha_k\rangle\,\frac{D_k}{Dt}\,\langle\langle p_k\rangle\rangle}_{\text{Pressure energy}} + \qquad (3) \\
&\underbrace{\frac{\xi_h}{A}\alpha_{kw}q''_{kw}}_{\text{Heat transfer in the wall}} + \underbrace{\langle\Gamma_k\rangle\,\langle\langle h_{ki}\rangle\rangle}_{\text{Change of phase energy}} + \underbrace{\langle a_i q''_{ki}\rangle}_{\text{Interfacial energy transfer}} + \underbrace{\cancelto{0}{\langle\Phi_k\rangle}}_{\text{Viscous dissipation}}
\end{aligned}
$$

Simplifications to (3) are:

1. The distribution parameter C_{hk} in (3) is approximated to 1 in the same way as the distribution parameter in the momentum equation.

2. The diffusive term is ignored due to small variations of fluid temperature in the axial direction compared with variations in the radial direction.

3. The viscous dissipation function is not important at low velocities.

The conservation equations need additional equations known as closure relations to be solved. These equations are necessary to find the next terms:

1. Interaction force between the two phases $\left\langle M_k^d\right\rangle$.

2. Wall friction $\dfrac{4\alpha_{kW}\tau_{kW}}{D}$

3. Heat transfer in the wall $\dfrac{\xi_h}{A}\alpha_{kw}q''_{kw}$

4. Energy interaction between the two phases $\langle a_i q''_{ki} \rangle$

The set of closure relations modifies significantly the performance of the simulation. In steady state, pressure drop depends only on the wall friction $\frac{4\alpha_{kW}\tau_{kW}}{D}$, and the interaction force between the two phases $\left\langle M_k^d \right\rangle$. Due to pipe size temperature gradients inside it can be neglected in the radial direction. Furthermore, it is small in the axial direction of the wellbore.

The conservation equations are used to compute the following variables:

1. Gas and liquid velocity (2 unknowns).

2. Void fraction (1 unknown).

3. Pressure (1 unknown).

4. Enthalpy of each fluid (2 unknowns).

The SIMPLE (Semi-Implicit Method for Pressure-Linked Equations) method will be used to solve pressure-velocity coupling in these equations. In [12] different approaches are explained to implement the SIMPLE method according to the way continuity equations are used. There are two categories: MCBA (mass conservation based algorithm) and GCBA (geometric conservation base algorithm). The MCBA uses the global continuity equation to find the pressure correction and the continuity equations of one phase to find the volume fraction of each phase. Instead, the GCBA uses the correction pressure to ensure that the sum of the volume fractions of the phases is equal to 1.

3 NUMERICAL METHODOLOGY

As a starting point for this work, some simplified and mechanistic models were implemented. Afterwards, a drift flux model was implemented and a two fluid model is now being implemented.

The simplified and mechanistic models have been implemented to compare predictions and they were also used to test the black oil model. The black oil model is necessary to predict fluid properties, i.e. density, solubility, volumetric factor, and viscosity. An example of the obtained results is presented in figure 72.

The proposed two fluid model is implemented in three stages. Firstly, a simplified two fluid model for water-air fluids (without black oil model) is applied in the water faucet problem to verify the correct implementation of the model against data published in the literature. The following hypothesis are considered:

1. Fluids: air-water.

2. Density and viscosity constant.

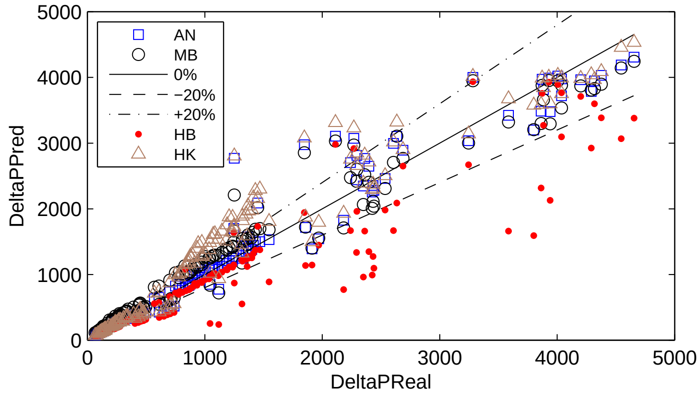


Figure 2: Comparison of measured pressure drop against values predicted by the Ansari et al, Mukherjee & Brill, Hagedorn & Brown and Hasan et al models. Results presented in the International Mechanical Engineering and Agricultural Sciences Congress 2013 [8].

Then, the two fluid model is solved in a steady or pseudo-steady state with data from oil wells to test the selected closure relations for frictions factors. Finally, the two fluid model is used to solve an unsteady-state to check the performance of the complete model proposed.

Previously, it was mentioned that the SIMPLE method is used to solve the pressure-velocity coupling. In the two fluid model, pressure and void fraction should be found from the conservative equations [12, 13]. In the present work, the pressure correction equation implemented (4) is deduced from the sum of the liquid and a scaled gas mass conservation equation. Furthermore, the liquid continuity equation is used to find the holdup and then, the void fraction. From (4) it is obtained the correction pressure value that mainly balance the gas phase continuity equation, and (5) calculates the void fraction to correct the imbalance of the liquid phase.

$$\begin{gathered}\tfrac{\partial}{\partial z}[(\zeta C_{\rho g}\alpha v_g^{m*}+C_{\rho l}h_l v_l^{m*})p'] = \tfrac{\partial}{\partial z}[(\zeta\rho_g\tfrac{\alpha^2}{a_p^{vg}}+\rho_l\tfrac{h_l^2}{a_p^{vl}})\tfrac{\partial p'}{\partial z}] - \tfrac{\partial(\rho_l h_l+\zeta\rho_g\alpha)}{\partial t} \\ -\tfrac{1}{z}(\zeta\rho_g\alpha v_g^{m*}+\rho_l h_l v_l^{m*})+(\zeta\Gamma_g+\Gamma_l) \\ \zeta=\tfrac{\rho_l}{\rho_g}\end{gathered} \tag{4}$$

$$\frac{\partial h_l\rho_l}{\partial t}+\frac{\partial}{\partial z}(h_l\rho_l v_l)=\Gamma_l \qquad h_l=1-\alpha \qquad \Gamma_l=\Gamma_g \tag{5}$$

Finally, it is worth to highligth that in order to reproduce a wellbore starting from a point where two phase flow is already present, it is neccesary to apply a new correlation to find the holdup at the inlet boundary.

4 BLACK OIL MODEL

The black oil model assumes that at a certain temperature, pressure, API gravity and gas gravity the crude oil has a fixed value of solubility of gas and an oil formation volume factor [2]. The crude oil and the gas are in a equilibrium condition all the time.

Boundary conditions in the oil industry are expressed with volumetric flow rate instead of velocities. Therefore, a different approach should be followed to calculate inlet velocity condition. The following equations are necessary to get the value of the volumetric flow rate in all control volumes. These equations depend on fluids properties:

$$v_{sl} = \frac{v_l}{1-\alpha} \qquad v_{sg} = \frac{v_g}{\alpha} \tag{6}$$

$$q_l = v_{sl} A_p \qquad q_g = v_{sg} A_p \tag{7}$$

$$q_{o,sc} = \frac{q_l f_o}{B_o} \qquad q_{g,sc} = \frac{q_g}{B_g} + q_{o,sc} R_s + \frac{B_o f_w}{B_w(1-f_w)} R_{sw} q_o sc \tag{8}$$

One of the most important fluid properties found in the black oil model is the gas solubility R_s. There are several correlations to find the value of R_s, e.g., Standing [2] proposes the following correlation: $R_s = \gamma_g[(\frac{p}{18.2} + 1.4)10^{0.0125\gamma_{API} - 0.00091T}]^{1.2048}$. Many properties of the crude are calculated based on R_s.

In (8) the following properties are also needed:

1. B_o is the oil formation volume factor: $B_o = 0.9759 + 0.00012[R_s(\frac{\gamma_g}{\gamma_o})^{0.5} + 1.25T]^{1.2}$
2. B_g is the gas formation volume factor and it depends of the compressibility factor, temperature and pressure. $B_g = 0.0283\frac{ZT}{p}$.
3. R_{sw} is the solubility of gas in water:

$$R_{sw} = A + Bp + Cp^2$$

$$A = 2.12 + 3.45e - 3T - 3.59e - 5T^2$$

$$B = 0.0107 - 5.26e - 5T + 1.48e - 7T^2$$

$$C = -8.75e - 7 + 3.9e - 9T - 1.02e - 11T^2$$

4. The oil fraction in a liquid phase is approximated (assuming non slippage) as: $f_o = q_o/(q_o + q_w)$ and the water fraction in a liquid phase is $f_w = q_w/(q_o + q_w)$.

5 MODEL VERIFICATION

The water faucet problem [13] is commonly used to test the two phase flow models, and it was selected in this work to verify the model behaviour against an analytic solution.

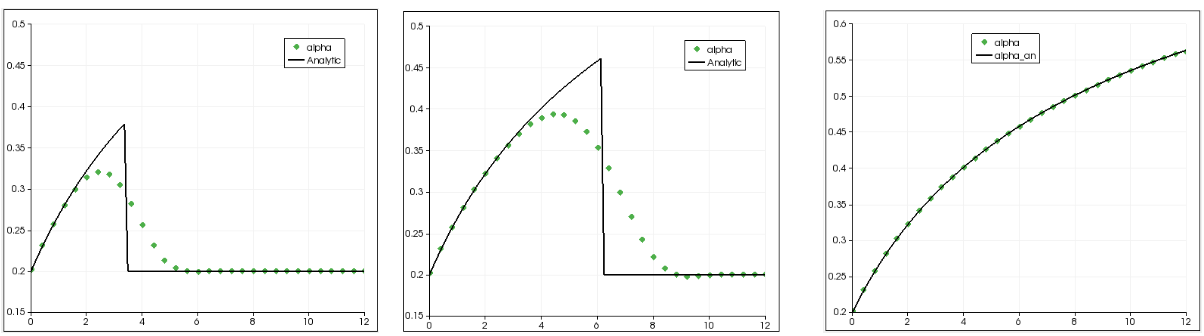
The faucet problem consists in a vertical pipe with a water (only liquid) entrance in the top of the pipe. The bottom hole of the pipe is open to the environment at atmospheric pressure. The water go out of the pipe and air go to the pipe by this hole. In the faucet problem it is assumed that the void fraction is constant along the pipe at the initial time.

The analytic solution neglects the viscosity of fluid and all friction forces. The void fraction and the liquid velocity as a function of time and space is shown in (9) and (10), respectively [14].

$$\alpha(x,t) = \begin{cases} 1 - \frac{\alpha_l^0 v_l^0}{\sqrt{2gx + (u_l^0)^2}} & if \quad x \leq u_l^0 t + \frac{1}{2}gt^2 \\ 1 - \alpha_l^0 & otherwise \end{cases} \tag{9}$$

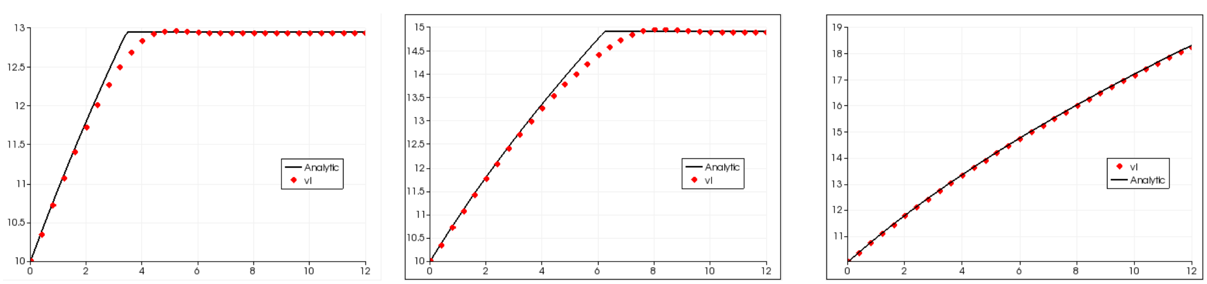
$$u_l(x,t) = \begin{cases} \sqrt{2gx + (u_l^0)^2} & if \quad x \leq u_l^0 t + \frac{1}{2}gt^2 \\ u_l^0 + gt & otherwise \end{cases} \tag{10}$$

Figure 3 shows the evolution of the void fraction at three different times, and Figure 4 shows liquid velocity at the same moments. Data were obtained for a 200 control volumen simulation with a time step of 0.01 [s]. The solution found is similar to the analytical solutions in the transiet stage, and almost the same when steady state is reached.



$t = 0.3[s]$ $t = 0.5[s]$ $t = 2[s]$

Figure 3: Comparison of the implemented model with an analytic solution of the void fraction.



$t = 0.3[s]$ $t = 0.5[s]$ $t = 2[s]$

Figure 4: Comparison of the implemented model with an analytic solution of the liquid velocity.

The simulation of the wellbore with the black oil model is being implemented, and only preliminar results have been obtained , therefore, final results are not included in this paper.

6 CONCLUSIONS

In this work it is shown a procedure to develop and implement a two fluid one dimensional model. Mass, momentum and energy conservation equations for each fluid are explained and simplified for the specific case of the two phase flow in wellbores. The model has been implemented in two stages. In the first one, the model considers the transient term and constant physical properties without flow patterns for air-water fluids. Comparison of the results with an analytical solution for the water faucet problem is done. A fair good agreement between calculated and anlytical solutions is observed for this configuration. Then, the black oil model is applied to evaluate physical properties and flow patterns are also taked into account.

Regarding pressure correction equation, it has been observed that when it is obtained from the global continuity equation using the scale factor for the gas continuity equation, instead of ignoring scale factor in conjuction with the liquid continuity equation to find the holdup improves convergence for the MCBA method.

ACKNOWLEDGMENTS

This work received financial support provided by VIE-CEC Vicerrectoría de Investigación y Extensión - Campo Escuela Colorado No. 8557, Universidad Industrial de Santander, and, Jóvenes Investigadores Colciencias No. 617.

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