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NUMERICAL SIMULATION OF STRUCTURAL BEHAVIOUR OF MEMBRANE RESTRAINED ELASTIC GRIDSHELLS

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Abstract. Developable elastic gridshells are cost-effective lightweight structures making use of a rapid construction process, in which the geometry of the gridshell is obtained by bending an initially flat grid. This particular shaping process saves time during the erection of the structure, as the grid rods must not be bent individually but the grid can be shaped as a whole. Moreover, the assembly of the connections between the superposed rod layers of the grid can be done on the ground on a flat geometry, which is easier than connecting single elements in the air. Nevertheless, in order to introduce shear stiffness to the initially unstable grid lattice, an additional layer of beam elements or diagonal cables must be added. The assembling of this bracing layer is usually time-consuming and requires additional supplies such as cherry-pickers or movable scaffolds. In this manner one of the great advantages - the rapid deployability of elastic gridshells - is clearly reduced. In order to accelerate the construction process of deployable elastic gridshells, we propose to use tensile membranes as restraining in addition to cladding elements. In this paper, the structural behaviour of a membrane restrained elastic hemispheric gridshell with different connection configurations between membrane and grid has been analysed by means of finite-element-methods and compared to that of a 1:1-prototype. The results show the capacity of the membrane to reduce the structure's deviations under asymmetric load. Furthermore, some constructive aspects observed during the construction of the prototype and having an influence on the bearing behaviour of the gridshell are presented and discussed.

1 INTRODUCTION

Developable elastic gridshells are cost-effective lightweight structures making use of a rapid construction process, in which the geometry of the gridshell is obtained by bending an initially flat grid. This particular erection process saves time and costs, as the grid profiles must not be individually bent but the grid can be shaped as a whole. Moreover, the connections between superposed grid profiles can be assembled on the ground in the initially flat geometry, which is easier than joining them in the air [1-4].

Nevertheless, after the bending process, the grid must be additionally braced in order to provide the structure with in-plane stiffness. Usually the restraining members consist of a third outer layer of diagonal profiles, tensile cables or rigid panels. Their assembling is generally more time-consuming than that of the grid, as the multiple bracing elements must be individually handled, connected in the air to the grid nodes and, in case of using diagonal profiles, bent into the gridshell geometry. In this manner one of the great advantages of elastic gridshells - its rapid construction through developable grids - is clearly reduced.

In order to optimise the construction process of developable elastic gridshells, we propose to employ a unique membrane surface to cover and at the same time to restrain the grid. Thus, fewer elements must be handled on site, less time is needed for the construction process and material savings are achieved.

The use of tensile membranes as restraining element has been already investigated in hybrid arch structures with stress-free curved and elastically-bent chords in [5-7]. Tensile restraining membranes can efficiently reduce deformations by hybrid structures. Their influence on the global bearing behaviour of the structures strongly depends on the material and sectional properties of the structural elements and stiffness, pre-stress and orientation of the membrane.

In this paper, the potential and limitations of using tensile membranes to restrain elastically-bent gridshells are investigated. The influence of the membrane and grid properties has been numerically analysed, firstly, on a 4-field grid and, secondly, on a hemispheric gridshell of 5m diameter.

2 MEMBRANE RESTRAINED 4-FIELD GRID

The restraining effect of tensile membranes and its dependence on the membrane properties have been firstly studied on a 2m x 2m 4-field grid. The shear stiffness of the 4-field grid, restrained with different elements and membranes with different properties, has been analysed by means of finite element modelling using the FEA-software Sofistik. The 4-field grid has been punctually loaded by applying horizontal forces on the upper corner; the resulting deformations have been compared.

2.1 Description of the structure

The 2m x 2m 4-field grid is composed of two superposed crosswise layers of three continuous profiles. The intersections between the layers take place every 1m. The material of the profiles corresponds to glass fibre reinforced plastics (E-Modulus = 25.10^3 N/mm^2), their section is tubular with a diameter of 20 mm and a thickness of 3 mm. The connections between superposed profiles allow scissoring - variation of the angle between profiles - of the grid fields.

The restraining effect of the tensile membrane has been compared with that of plywood panels, diagonal steel cables and GFRP profiles (Figure 1). All the restraining and bracing elements are connected punctually to the grid intersection points. The membrane and plywood panels have been modelled as plane elements, the cables as tensile elements and the profiles as continuous beam elements. In the case of the membrane and plywood panels, contact between panels and grid has been modelled using spring elements.

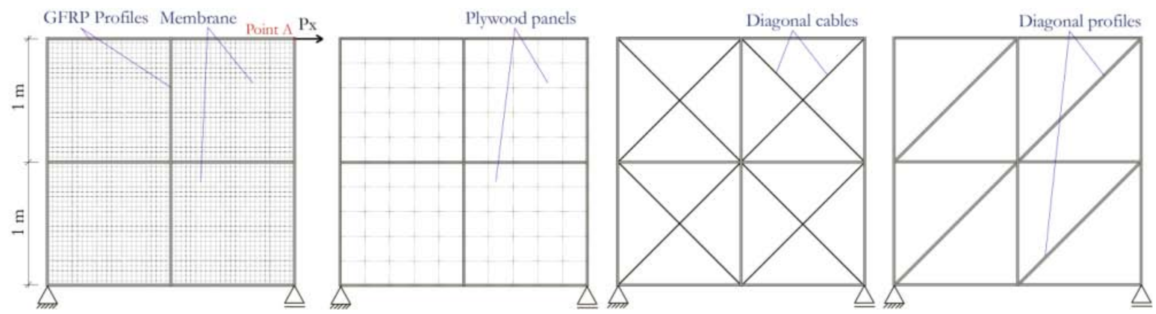

Figure 1: Structural system of the 4-field grid with different restraining elements

In the following paragraphs the studied parameters and properties of the restraining elements are summarised:

1. Membrane's orientation

Tensile membranes are usually strongly elastically anisotropic. The influence of the membrane's orientation - orientation of yarn's fibres - has been analysed by comparing the shear stiffness of two grids, restrained with membranes whose warp and weft yarns are oriented diagonally and longitudinally.

Table 1: Element properties of the models analysing the influence of membrane's orientation

Restraining element	*Material*	*Warp/weft orientation*
1.a Membrane	*Polyester cloth, PVC/PVDF coating* *Type III, thickness = 1.02 mm* *Prestress $_{warp/weft}$ = 0.1 kN/m*	*Diagonal*
1.b Membrane	*E $_{warp/weft}$ = 1000 N/mm2 (1020 kN/m)* *Shear modulus G = 30 N/mm2 (30 kN/m)* *Poisson's ratio ν= 0.25*	*Longitudinal*

2. Membrane's stiffness

Membranes dispose of lower axial stiffness than diagonal cables, bars or rigid panels and can only carry in-plane forces so that with them only the shear stiffness of the grid will be improved. The mechanical properties of tensile membranes strongly vary depending on the nature of their components (yarn and coating). In order to investigate the influence of the membrane's stiffness on the bearing capacity of the 4-field grid, membranes with varying values of elasticity modulus have been modelled and their restraining effect has been compared with that of bracing diagonal cables, beams and rigid plane elements.

Table 2: Element properties of the models analysing the influence of membrane's stiffness

Restraining element	*Material*	*Stiffness*
2.a Membrane	*Polyester cloth, PVC/PVDF coating, Typ III, t =1.02 mm Prestress $_{warp/weft}$ = 0.1 kN/m G = 30 N/mm2 (30 kN/m) Poisson's ratio v= 0.25*	*$E_{warp/weft}$ = 500 N/mm^2 (510kN/m)*
2.b Membrane		*$E_{warp/weft}$ = 1000 N/mm^2 (1020kN/m)*
2.c Membrane		*$E_{warp/weft}$ = 1500 N/mm^2 (1530kN/m)*
2.d Membrane		*$E_{warp/weft}$ = 2000 N/mm^2 (2040kN/m)*
2.e Cables	*Stainless steel, d = 6 mm*	*E = 130.10^3 N/mm^2*
2.f Profiles	*GFRP, d = 20mm, t = 3 mm*	*E = 25.10^3 N/mm^2*
2.g Panel	*Plywood F40/40 E60/40, t = 4mm*	*$E_\parallel$=4400 N/mm^2, $E_\perp$=4700 N/mm^2*

3. <u>Membrane's prestress</u>

Membrane surfaces represent light structural elements as they are supposed to carry only tensile forces. Loss of stress, and with it wrinkles on the membrane, due to compression forces should be avoided. By inducing tensile prestress the stiffness of the membrane can be significantly increased. In the case of the 4-field grid, four models with different prestress levels have been defined and the distribution of the principal membrane forces as well as the shear stiffness of the grid have been analysed.

Table 3: Element properties of the models analysing the influence of membrane's prestress

Restraining element	Material	Prestress
3.a Membrane	*Polyester cloth, PVC/PVDF coating Typ III, thickness = 1.02 mm Pre-stress warp/weft = 0.1 kN/m Ewarp/weft = 1000 N/mm^2 (1020 kN/m) Shear modulus G = 30 N/mm^2 (30 kN/m) Poisson's ratio v= 0.25*	*$v_{warp,weft}$= 0.1 kN/m*
3.b Membrane		*$v_{warp,weft}$= 1.0 kN/m*
3.c Membrane		*$v_{warp,weft}$= 1.5 kN/m*
3.d Membrane		*$v_{warp,weft}$= 2.0 kN/m*

2.2 Results

2.2.1 Influence of membrane's orientation

The following graphic shows the nodal displacement (Figure 1) by increasing horizontal load at Point A on grids restrained with membranes with varying orientation of warp and weft yarns. One can observe the optimisation of the shear stiffness of the grid by orientating warp and weft in diagonal direction: by a horizontal load of 0.55 kN the nodal displacement at Point A results on 175 mm with a longitudinal and 18 mm with a diagonal orientation of the fibres. Figure 3 illustrates the distribution of the principal membrane forces for both warp/weft orientations and shows the higher efficiency of the diagonal orientation.

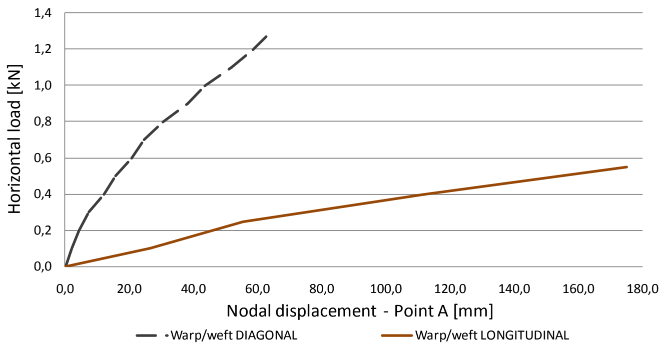

Figure 2: Nodal displacement by increasing horizontal point load at Point A on grids restrained with membranes with diagonal (blue) and longitudinal (red) orientation of the warp/weft yarns

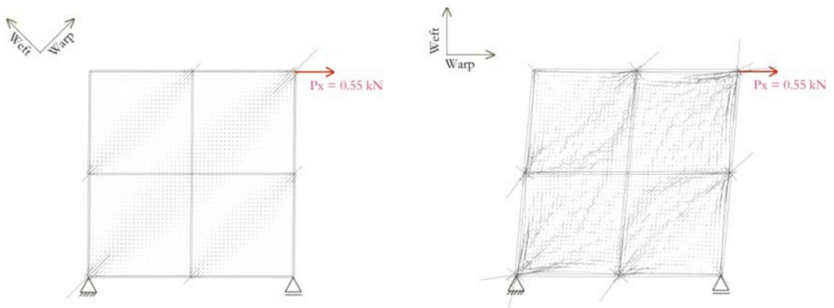

Figure 3: Distribution of principal membrane forces and deformation under horizontal load of 0.55 kN at Point A on the grid restrained with membranes with diagonal (left) and longitudinal (right) orientation of warp/weft

2.2.2 Influence of membrane's stiffness

On the following graphic, the nodal displacement of the grid restrained with membranes of varying stiffness ($E_{warp/weft}$ = 500 – 2000 kN/m^2) under increasing horizontal load at Point A is illustrated and compared to that of grids restrained with diagonal cables, diagonal GFRP-tubes and F40/40-plywood panels. One can observe that the membrane's stiffness has a strong influence on the deformations of the grid. By a horizontal load of 1.15 kN, the nodal displacement at Point A corresponds to 113 mm, 55 mm, 39 mm and 28 mm for the membranes with elasticity moduli of 500 N/mm^2, 1000 N/mm^2, 1500 N/mm^2 and 2000 N/mm^2, respectively. Nevertheless, the restraining effect of the tensile membrane remains significantly lower than that of conventional bracing elements.

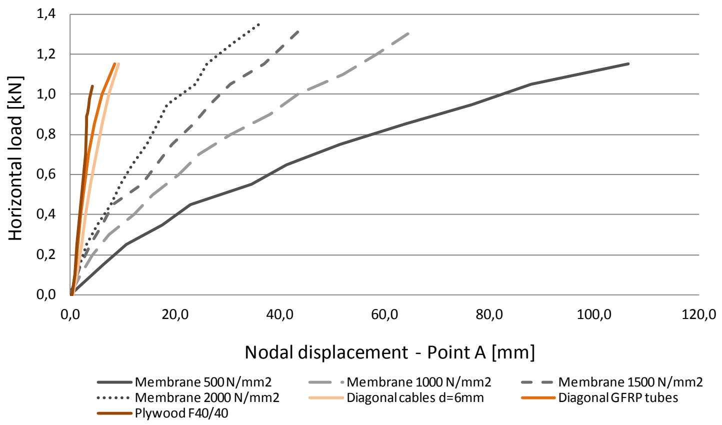

Figure 4: Comparison of restraining effect of tensile membranes of different stiffness, diagonal cables, diagonal GFRP tubes and plywood panels by the deformation of the 4-field grid under increasing horizontal point load

The final choice of the membrane's stiffness should however consider its constructive consequences. The higher the stiffness is, the more complex its confection and handling on site will be. Figure 5 (left to right) illustrates the deformation of the 4-field grid under a horizontal point load in A of 1.15 kN, restrained with membranes of $E = 500$ N/mm^2 and $E = 2000$ N/mm^2 (exaggeration's factor of 2) and with diagonal cables, bars and plywood panels (exaggeration's factor of 5).

Figure 5: Deformation of 4-field grid restrained with membranes with $E = 500$ N/mm^2 and $E = 2000$ N/mm^2 (exaggerated by a factor of 2), diagonal cables, bars and plywood panels (exaggerated by a factor of 5) under a horizontal load of 1.15 kN at Point A

2.2.3 Influence of membrane's prestress

The following figure illustrates the nodal displacement of the 4-field grid restrained with membranes with varying prestress level ($v_{warp/weft}$ = 0.1, 1.0, 1.5 and 2.0 kN/m) under increasing horizontal load at point A. For prestress levels of 1.0, 1.5 and 2.0 kN/m, one can observe a kink on the curves at horizontal loads of 0.5, 0.7 and 0.9 kN, respectively. This kink corresponds to the loss of tensile stress on the diagonal receiving compression forces. Inducing higher prestress on the membrane, the deviation at point A could be considerably reduced, for example from 64 mm with $v = 0.1$ kN/m to 40 mm with $v = 2.0$ kN/m, under a horizontal load of 1.3 kN.

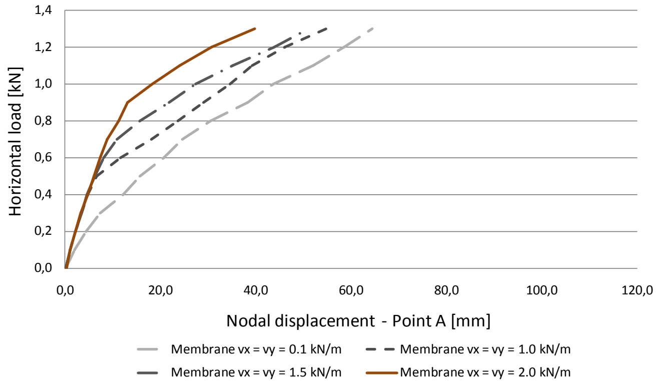

Figure 6: Nodal displacement by increasing horizontal load at point A in grids restrained with membranes with prestress levels in warp and weft of 0.1, 1.0, 1.5 and 2.0 kN/m

Figure 7 shows the principal membrane forces of the restraining membranes with 0.1 and 2.0 kN/m prestress by horizontal loads of 0.2, 0.8 and 1.3 kN. One can observe that, with a higher prestress level, the membrane is also able to carry forces on the diagonal subjected to compression until the tensile stresses are consumed.

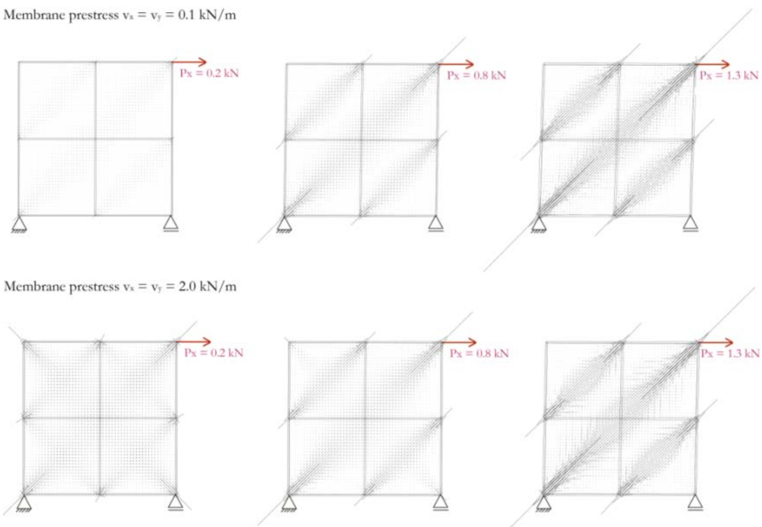

Figure 7: Principal membrane forces of restraining membranes with 0.1 (top) and 2.0 kN/m (bottom) prestress for horizontal loads of 0.2, 0.8 and 1.3 kN

3 MEMBRANE RESTRAINED ELASTIC GRIDSHELL – HYBRID HEMISPHERE

The aim of this chapter is to investigate the restraining effect of tensile membranes on doubly-curved elastic gridshells. A hemispheric regular gridshell of 5 m diameter and 0.74 m mesh size has been considered as example. The analyses focus on the influence of the connection conditions between grid and membrane: with and without joining at the grid nodes. The structural analysis has been performed using three-dimensional non-linear finite element models defined with the FEM-package of SOFISTIK.

3.1 Description of FEM-Model

Grid profiles have been modelled as beam elements and the membrane as plane elements. The grid profiles are made of glass fibre reinforced plastic (E=25000 N/mm^2, d= 20 mm, t= 3 mm), the membrane corresponds to a Ferrari Précontraint 1302 S2 with polyester cloth and PVC/PVDF coating ($E_{warp/weft}$= 1500/1200 N/mm^2, t= 1.02 mm). The connections between superposed layers have been modelled as hinged couplings.

Two types of spring elements have been used for the connection between grid and membrane. The first type corresponds to springs with only axial stiffness, modelling the contact of the membrane over the grid. They are defined along all the beam elements, their direction is approximately perpendicular to the membrane surface, their coefficient is 10 kN/m and they can only carry compression forces. The second type corresponds to the connection of the membrane at the grid nodes. They are defined only at the intersection of the grid layers and their direction is also approximately perpendicular to the membrane surface. They have axial and lateral stiffness with coefficients of 10^3 kN/m. Fixed supports have been defined at the profiles' ends. The edge support of the membrane is also modelled using spring elements with axial and transversal coefficients of 1 kN/m. The spring coefficients of the connections have been defined after benchmarking the FE-model with the prototype described in Chapter 4.

The FEM consists of two parts. Firstly, the bending process of the grid has been modelled. The bending of the beam elements has been generated applying virtual cable elements, inducing the deformation of the profiles, as suggested by Lienhard et al. in 2011 [8]. Once the bent geometry of the grid has been simulated, the membrane and connections to the grid can be activated and the loading of the structure can be modelled.

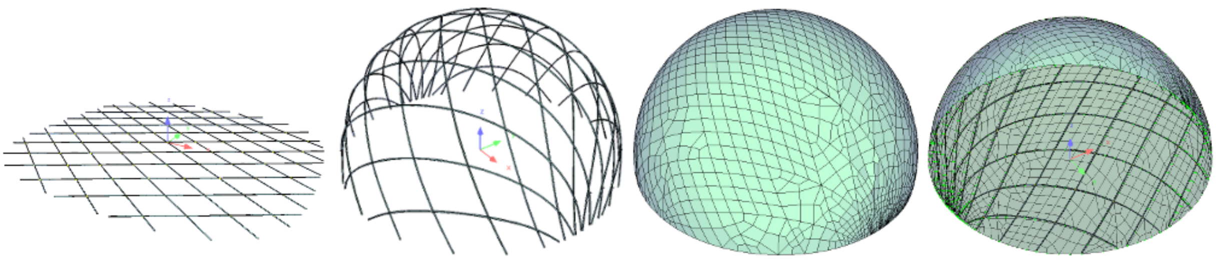

Figure 8: FEM-Model of hybrid elastic gridshell; (from left to right) planar grid's geometry, bent grid's geometry; hybrid gridshell from the outside and from the inside

3.2 Study of the influence of connection between grid and membrane

The connection of the membrane to the grid has important consequences on the construction process of the gridshell. Therefore, it is important to be able to estimate its structural effects. In this chapter, the structural behaviour of the grid after its shaping process and under asymmetric loading has been calculated and compared for three systems: grid without membrane, grid with membrane without joining to the grid and grid with membrane with joining to the grid at grid nodes.

3.2.1 Influence on geometry resulting from bending process

Once the grid has been bent in a specific geometry and fixed at its edges, if the applied shaping forces are removed before restraining the grid, it adopts a new equilibrium and a new geometry. Figure 9 illustrates the nodal displacement of the grid when the external forces are removed for the three configurations. One can see that using the membrane as restraining element, the maximum nodal displacements could be reduced from 50 mm to 24 mm and 19 mm, without and with joining to the grid.

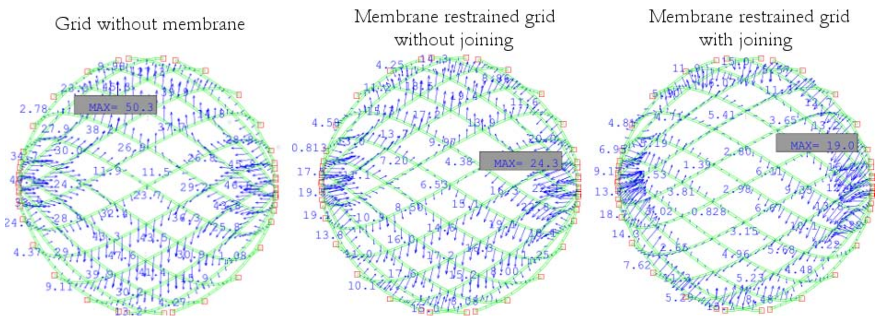

Figure 9: Nodal displacements of the grid by removing external shaping forces

3.2.2 Influence on bearing behaviour under asymmetric load

The restraining effect of the membrane and the influence of the connection to the grid are more evident under the action of external forces. Half of the hemispheric grid has been loaded with vertical loads of 0.2 kN in 47 intersection points (Total load = 9.4 kN, Projected area = 9.8 m^2). Figure 10 shows the respective grid deformations. The restraining effect of membrane achieves to reduce the maximum nodal displacements of the grid from 189 mm to 90 and 30 mm, without and with joining to the grid.

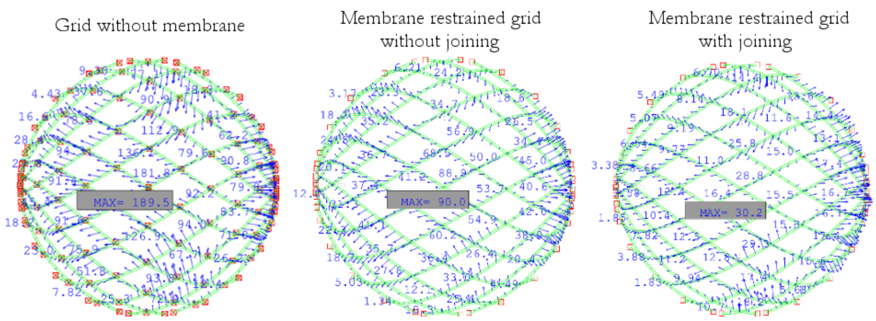

Figure 10: Nodal displacements of the grid under asymmetric load

4 EXPERIMENTAL VALIDATION OF NUMERICAL SIMULATION

The purpose of the prototype is to benchmark the numerical simulation and to identify constructive aspects difficult to be implemented by the construction of membrane restrained elastic gridshells.

4.1 Construction

The elements of the prototype correspond to those of the FE-model defined in Chapter 3: grid composed of glass fibre-reinforced plastic profiles of 20 mm diameter and 3 mm wall thickness and covering membrane Ferrari Précontraint 1302 S2 with polyester cloth and PVC/PVDF coating. Double swivelling clamps with inner rubber walls have been used to connect the superposed grid layers. The prototype has been tested with and without connection between membrane and grid. On the first case, this connection was provided by threaded pins screwed on nuts welded on the upper clamps of the grid nodes. For it, the membrane was perforated with eyelets and reinforced at 85 positions, corresponding to the grid intersection nodes. The grid was fixed on a ring made of OSB-plates using single clamps. To fix the membrane, a rope was laced through the edge eyelets and a PVC-ring (Figure 11).

The construction of the hybrid gridshell started with the assembling and afterwards bending of the initially plane grid by pushing it upwards and connecting the profiles' ends to the ring. Then, the membrane was progressively spread over the grid and finally tensed and fixed to the ring. The hybrid gridshell was loaded with weight loads of 10.3 – 18.9 kg applied on one half of its surface (asymmetric load). The deformations were registered using 3D-scanning.

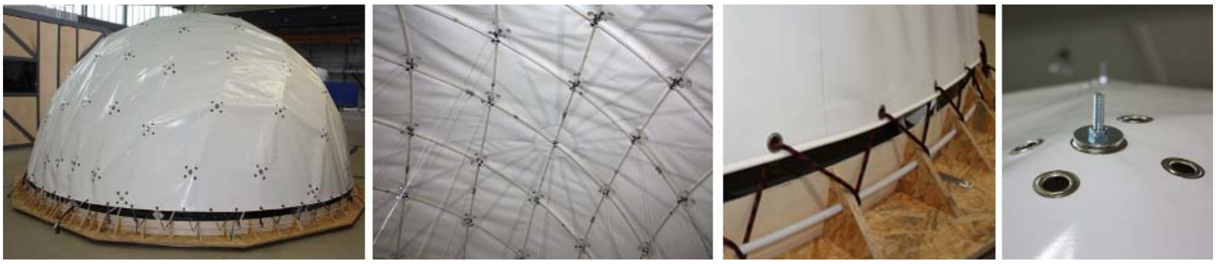

Figure 11: Prototype of the hybrid hemisphere of 5 m diameter: outer and inner view, details at edge ring and grid nodes with joining to the membrane

4.2 Results

By comparing the grid deformations of the numerical and physical models, it was noticed that the stiffness of the grid nodes has an important influence on the grid structural behaviour. Therefore, the coupling elements were replaced by spring elements with axial and lateral coefficients of 10 and 1 kN/m, respectively. In the case where the membrane was connected to the grid, significant wrinkles appeared on the membrane surface (Figure 11), probably due to imperfections on the connections (e.g. sliding of clamps), which results on high differences between prototype and FE-model. In the case where the membrane was not connected to the grid, wrinkles could be avoided by tensioning the membrane with the edge rope. This last case is going to be considered for the comparison of the grid deformations under asymmetric loading.

Figure 12 compares the nodal displacements of the prototype and finite-element-model at 74 registered points. One can observe similar distributions; nevertheless, the deviations of the prototype are to some extent higher than those of the FE-model. Moreover, differences between both systems are lower on the middle profiles (e.g. number 5 – 7) than on the profiles at the sides (e.g. 1 and 11). That can be due to the modelling of the contact between membrane and grid or to the modelled membrane's geometry.

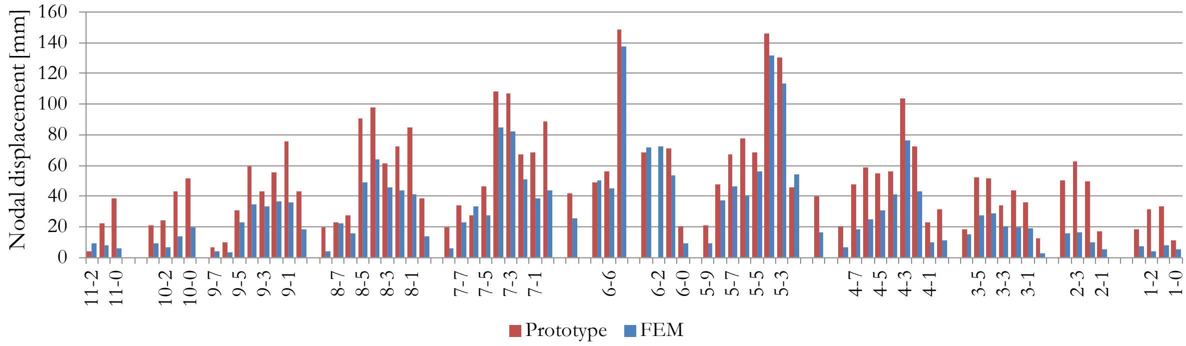

Figure 12: Comparison between prototype (red) and finite element model (blue) of nodal displacements at 74 grid nodes under asymmetric loading

5 CONCLUSION

This paper has the aim to investigate the potential of tensile membranes as restraining element for elastic gridshells. The structural effect of the membrane on the shear stiffness of a 4-field grid and on the bending geometry and bearing capacity of a hemispheric gridshell has been analysed by means of finite-element-modelling. An important increment of the stiffness of the grids could be observed when restraining them with tensile membranes.

To benchmark the finite-element-model, a prototype of the hybrid hemisphere with 5 m diameter has been built and loaded and the resulting nodal displacements have been compared to those of the numerical model. By the construction of the hybrid prototype, technical difficulties were identified: e.g. achievement of wrinkles-free membrane when joining it to the grid. Furthermore, the benchmarking of the FE-model with the prototype allowed determining the influence of the properties of the connection elements on the bearing behaviour of the hybrid structure.

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