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DIRECT PROCEDURE FOR THE DETERMINATION OF CONVENTIONAL MODES WITHIN THE GBT APPROACH

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Abstract. This paper presents a procedure for the determination of the conventional modes to be used in the framework of the Generalised Beam Theory (GBT) for the analysis of thin-walled members. These deformation modes consist of the rigid, distortional, local and Bredt shear modes and, with the proposed approach, are evaluated in a single step cross-sectional analysis. This procedure is applicable to any type of cross-section, i.e. open, closed and partially-closed one. The algorithm differs from that of the classical GBT, which requires a two-step evaluation procedure, consisting of an initial choice of the vector basis and its successive orthogonalization. The procedure is based on the definition of a quadratic functional, whose steady condition leads to an eigenvalue problem, and directly generates the sought orthogonal basis, here found using a finite element analysis. The accuracy of the proposed method is validated by means of a numerical example carried out with a partially-closed section. It is shown that the conventional modes derived with the proposed approach are identical to those determined with the classical two-step procedure.

1 INTRODUCTION

The Generalised Beam Theory (GBT) is a method of analysis widely used for thin-walled members (TWM). With this approach, TWMs are considered as an assembly of thin plates, free to bend in the plane orthogonal to the member axis. GBT differs from the classical Vlasov theory [1] because it is able to account for the deformability of the cross-section. In the spirit of the Kantorovich's semi-variational method, GBT transforms a three-dimensional continuous problem into a vector-valued one-dimensional problem. This is achieved by representing the displacement field of the TWMs as a linear combination of assumed deformation modes, defined at the cross-section in terms of in-plane and warping components, and amplitude functions describing how these modes vary along the member length. In this context, the use of GBT can be subdivided into two stages: a cross-sectional analysis where the deformation modes relevant to a particular thin-walled section are evaluated and selected, and a member analysis which makes use of these modes to determine the overall structural response defined in terms of the amplitude functions. This falls within the Kantorovich's semi-variational method [2], in which the dimensionality of a problem is reduced by the use of partially-assumed modes. The fundamental task to be performed for adequate modelling with the GBT consists of the identification of an appropriate and suitable set of deformation modes.

The GBT approach was first proposed by Schardt [3], who applied it to perform geometrically linear analyses (first-order GBT) and linearised stability analyses (second-order GBT). In the 1990's, Davies and co-workers reported a number of studies on the linear-elastic and buckling behaviour of thin-walled members, e.g. [4]. Extensive work in this area has been carried out over the last decade by Camotin and his co-workers, who extended the applicability of this approach to a wide range of applications, e.g. [5-8]. The simplification in the description of the displacement field, enabled within the GBT with the use of cross-sectional deformation modes and amplitude functions, was applied in recent years to the analyses carried out with the finite strip method [9] and the finite element method [10] to reduce the number of freedoms required in the modeling. In the classical GBT, the cross-sectional analysis was carried out in two steps, one to produce a first set of non-orthogonal modes, followed by a second one to make these modes mutually orthogonal. Orthogonalization was found to be convenient since the modes of the initial set were localised in nature, i.e. they involved a distortion/warping of a localised part of the section only. In fact, the orthogonal modes provided a global representation of the cross-sectional deformations and enabled the distinction among flexural or torsional rigid-cross-section modes, distortional modes (the cross-section deforms in its plane and warps, while nodes connecting its plate elements experience relative motion), or local modes (the plates bend in the cross-section plane, by leaving the nodes connecting them undisplaced, without cross-section warping).

In very recent years, attention has been devoted to generalise Vlasov's classic hypotheses of inextensibility and shear undeformability on which GBT was initially based, e.g. [11-12]. In references [13-15], starting from classical GBT kinematic assumptions, a semi-discretized approach was proposed leading to a modified GBT formulation, without the need of the two stages described above (cross-sectional and member analysis). With this approach, an eigenvalue problem was solved for complex eigenvalues and eigenvectors as in a non-proportionally damped dynamic analysis.

A different approach was presented in references [16-18], which proposed to use the dynamic eigenmodes of the cross-section, considered as a free planar frame, as the planar component of the conventional modes. These modes were successively supplemented by a warping component, determined via the Vlasov unshearability condition, for open cross-section, or the Bredt constant shear-flow condition, for closed cross-section. With this approach, the classical philosophy of GBT, consisting first in determining the warping and then in-plane components, was reversed. The deformation modes obtained from the dynamic approach did not coincide with those of the classical approach, since, for example, some of them were linear combinations of rigid and local modes, as a consequence of the fact that a dynamic eigenmode requires the vanishing of the integral of the inertia forces and, therefore, of the displacements, in the constant thickness case.

In this context, the present paper presents a one-step procedure for the evaluation of an orthogonal basis for the conventional modes based on the dynamic approach. [17] For the purpose of this study, the conventional modes include, consistently with the definition provided in reference [19], the rigid-body modes, the distortional ones, the local (bending) ones complemented, when dealing with closed sections, with a shear mode related to the case of pure torsional shear flow.

The basic idea of the paper relies on the fact that, according to the semi-variational method, a complete basis of deformation modes needs to be identified. This can be chosen as formed by the eigenvectors of *any* positive definite eigenvalue problem defined on the domain of the cross-section, provided the warping components are made *slaves* of the in-plane displacement components. For this purpose, a new functional is defined, with warping expressed in terms of the in-plane components, regardless of its physical meaning, whose stationarity condition identifies the eigenvalue problem generating the sought basis of modes. The continuous boundary value problem, however, is not solved exactly but, as usually carried out within the GBT approach, is implemented by means of a numerical method, discretising the cross-section with the finite element approach. Solution to the eigenvalue problem provides the profiles of the in-plane deformation modes, the corresponding warping terms being easily calculated in the post-processing stage.

2. PROPOSED GBT APPROACH

2.1 Overview of the GBT analysis

A generic thin-walled-member is considered as an assembly of thin flat plates which can form open, closed or partially-closed cross-sections. The displacement $\mathbf{u}(s,z)$ of an arbitrary point $P(s,z)$ lying in the mid-plane of the section thickness is described by:

$$\mathbf{u}(s,z) = u(s,z)\mathbf{e}_s(s) + v(s,z)\mathbf{e}_y(s) + w(s,z)\mathbf{e}_z(s) \tag{1}$$

where s is the curvilinear abscissa (if necessary defined on several branches) along the section mid-line C, z is the coordinate along the member axis, $\mathbf{e}_s(s)$, $\mathbf{e}_y(s)$ and $\mathbf{e}_z(s)$ are unit vectors in the tangential, normal and bi-normal directions at the abscissa s, respectively, and $u(s,z)$, $v(s,z)$ and $w(s,z)$ are the corresponding displacement components. The latter are expressed, in the framework of the classical GBT, as a linear combination of K assumed deformation modes and corresponding amplitude functions:

$$u(s,z) = \sum_{k=1}^{K} U_k(s)\varphi_k(z) \tag{2a}$$

$$v(s,z) = \sum_{k=1}^{K} V_k(s)\varphi_k(z) \tag{2b}$$

$$w(s,z)=\sum_{k=1}^{K}W_k(s)\varphi_{k,z}(z) \tag{2c}$$

where $U_k(s)$, $V_k(s)$ and $W_k(s)$ are the three components of the k-th deformation mode defined in terms of the abscissa s, $\varphi_k(z)$ depict the amplitudes of the k-th mode along the member axis z, and the comma denotes differentiation with respect to the variable that follows. The displacement field $\mathbf{d}(s,y,z) = d_s(s,z)\mathbf{e}_s(s) + d_y(s,z)\mathbf{e}_y(s) + d_z(s,z)\mathbf{e}_z(s)$ of an arbitrary point $Q(s,y,z)$ located within the thickness of the plate segments is evaluated according to the Kirchhoff plate model as:

$$\mathbf{d}(s,y,z)=\begin{bmatrix}\sum_{k=1}^{K}\left[U_k(s)-yV_{k,s}(s)\right]\varphi_k(z)\\ \sum_{k=1}^{K}V_k(s)\varphi_k(z)\\ \sum_{k=1}^{K}\left[W_k(s)-yV_k(s)\right]\varphi_{k,z}(z)\end{bmatrix} \tag{3}$$

from which the infinitesimal strain field $\boldsymbol{\varepsilon}$ can be expressed as:

$$\boldsymbol{\varepsilon}=\boldsymbol{\varepsilon}^M+\boldsymbol{\varepsilon}^F \tag{4}$$

where the membrane strain terms $\boldsymbol{\varepsilon}^M=(\varepsilon_s^M,\varepsilon_z^M,\gamma_{zs}^M)^{\mathrm{T}}$, relevant to the $y=0$ plane, are separated from the flexural strain components $\boldsymbol{\varepsilon}^F=(\varepsilon_s^F,\varepsilon_z^F,\gamma_{zs}^F)^{\mathrm{T}}$.

The weak form of the equilibrium can be derived by means of the principle of virtual work, from which the governing system of ordinary differential equations, expressed in terms of unknown amplitude functions, can be obtained from standard steps of variational calculus as:

$$\mathbf{C}\boldsymbol{\varphi}_{,zzzz}+\mathbf{D}\boldsymbol{\varphi}_{,zz}+\mathbf{B}\boldsymbol{\varphi}=\mathbf{p} \tag{5}$$

and relevant boundary conditions at $z = 0,L$:

$$\delta\boldsymbol{\varphi}_{,z}^{\mathrm{T}}\left(\mathbf{C}\boldsymbol{\varphi}_{,zz}+\mathbf{D}^a\boldsymbol{\varphi}\right)=0 \tag{6a}$$

$$\delta\boldsymbol{\varphi}^{\mathrm{T}}\left[\mathbf{C}\boldsymbol{\varphi}_{,zzz}+\left(\mathbf{D}^a-\mathbf{D}^b\right)\boldsymbol{\varphi}_{,z}-\mathbf{P}^z\right]=0 \tag{6b}$$

where **C**, **D** and **B** are $K\times K$ symmetric matrices defining the geometric and material properties of the cross-section, while the loading terms are collected in vectors **p** and $\mathbf{P}^z$. All terms are defined in reference [17].

2.2 Quadratic functional

A quadratic functional is defined, whose stationary condition provides a homogeneous self-adjoint problem, and the relevant eigenfunctions are taken as the deformation modes to be used in the GBT analysis. The functional is expressed as:

$$\mathrm{E}\left[V(s),W(s)\right] := \mathrm{E}_t\left[V(s)\right] - \lambda\left\{\mathrm{E}_l\left[V(s)\right] + \mathrm{E}_m\left[W(s)\right]\right\} \tag{7}$$

and

$$\mathrm{E}_t\left[V(s)\right] := \frac{1}{2}E_f\int_{\mathrm{C}} V_{,ss}^2(s)\,\mathrm{d}s \tag{8a}$$

$$\mathrm{E}_l\left[V_k(s)\right] := \frac{1}{2}E_f\int_{\mathrm{C}} V^2(s)\,\mathrm{d}s \tag{8b}$$

$$\mathrm{E}_m\left[W(s)\right] =: \frac{1}{2}E_a\int_{\mathrm{C}} W^2(s)\,\mathrm{d}s \tag{8c}$$

where the three contributions are proportional to the elastic energies of a segment of beam of unitary length. In particular, E_t represents the energy for bending in the direction transversal to the beam axis (i.e. in-plane bending), λE_l describes the elastic energy for longitudinal bending (i.e parallel to the beam axis), and λE_m is the energy for axial strain, caused by out-of-plane displacements. The objective is to evaluate the sets of functions $\{V(s),W(s)\}$ which render E stationary. The variation of the total functional E can then concisely be expressed in terms of the unknown eigenvectors **q** as:

$$\delta\mathrm{E}\left[V(s_e),W(s_e)\right] = \delta\mathbf{q}^{\mathrm{T}}\left[\mathbf{K}_V - \lambda\left(\mathbf{M}_V + \mathbf{M}_U\right)\right]\mathbf{q} = 0 \tag{9}$$

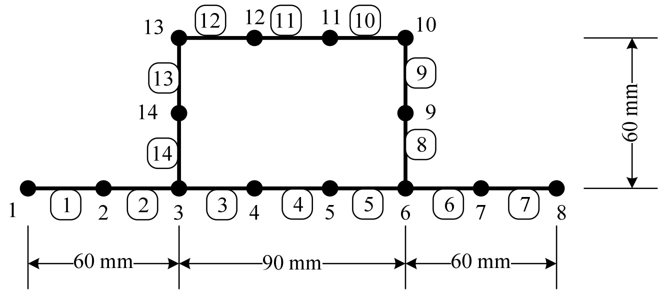
leading to the algebraic eigenvalue problem:

$$\left[\mathbf{K}_V - \lambda\left(\mathbf{M}_V + \mathbf{M}_U\right)\right]\mathbf{q} = 0 \tag{10}$$

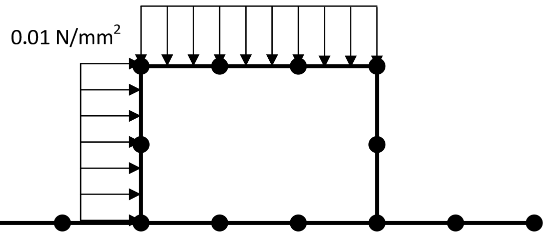
where $\mathbf{K}_V$, $\mathbf{M}_V$ and $\mathbf{M}_U$ are symmetric real matrices defined in reference [17]. Before the eigenvectors **q** are determined, internal constraints are applied to each element forming the cross-section to ensure their inextensibility. The deformation modes are then described by the eigenvectors calculated from the restrained eigenvalue problem. This problem is conceptually similar to the free vibration response of undamped structures in linear dynamics. In particular, this solution coincides with the eigenvalue problem of a free planar frame, in which $\mathbf{K}_V$ play the role of the classic stiffness and the mass matrix is composed by two parts, the former associated with the in-plane transversal inertia ($\mathbf{M}_V$) and the latter corresponding to the out-of-plane warping inertia ($\mathbf{M}_U$).

3 NUMERICAL EXAMPLE

A numerical example is presented in the following to validate the ability of the proposed approach (i) to produce a set of orthogonal conventional modes for open, closed and partially-closed sections and (ii) to compare these modes with those obtained with other available procedures. This is carried out for the partially-closed section illustrated in Figure 1. The comparison is first performed based on the results obtained from the cross-sectional analysis, followed by those calculated from the member analysis. For clarity, these are presented separately in the following.



(a) geometry and discretisation
for the partially-closed section



(b) loading condition
for the partially-closed section

Figure 1: Cross-sectional geometry and loading condition used in the numerical application

3.1 Cross-sectional analysis

The evaluation of the conventional modes for the cross-sections depicted in Figure 1 is carried out deriving them using three approaches: (i) the proposed functional and its algebraic eigenvalue problem; (ii) the dynamic approach presented in [16], and (iii) the two-step procedure used within the framework of classical GBT, e.g. [3,5]. In the latter case, the first step is carried out using the dynamic approach proposed in reference [16], then complemented with the additional eigenvalue problem used in the conventional GBT. This two-step procedure is equivalent to the classical GBT technique, which first evaluates local-type modes, e.g. [3,5], followed, in a second step, by their orthogonalisation. The accuracy of this two-step procedure has been verified against the results obtained with the GBTUL program [20], which enables the analysis of open sections, and the calculated values have shown to perfectly match.

The partially-closed section has been discretised as shown in Figure 1a because the interest is to evaluate the accuracy of the proposed procedure, and the level of accuracy could be easily improved introducing finer levels of mesh (i.e. higher number of elements). The conventional modes calculated for this cross-section are illustrated in Figure 2. For each mode, both in-plane and warping displacements have been presented. From these graphs, it is possible to observe that the proposed methodology perfectly matches the deformation modes of the two-step procedure, confirming its ability to produce orthogonal modes in one-step analysis. From the comparisons, it becomes apparent that the dynamic and the proposed (or two-step classical GBT) approaches produce a very similar set of modes, with the only differences being related to the nodal displacements present in some modes obtained by the dynamic analysis. The dynamic modes do not coincide with those of the proposed (or two-step classical GBT) approach, since some of these are linear combinations of rigid and local/distortional modes. This is a consequence of the fact that a dynamic eigenmode requires the vanishing of the integral of the inertia forces and, therefore, of the displacements in the constant thickness case.

3.2 Member analysis

The structural configuration adopted for the member analysis is based on a simply-supported static layout. The member end sections are assumed to be torsionally restrained and to be allowed to warp. The beam has a length of 1000 mm and a section thickness of 1 mm. The applied load is assumed to vary harmonically based on:

$$f_s = f_{sn} \sin\left(\frac{n\pi z}{L}\right) \quad ; \quad f_y = f_{yn} \sin\left(\frac{n\pi z}{L}\right) \quad ; \quad f_z = f_{zn} \cos\left(\frac{n\pi z}{L}\right) \qquad \text{(11a-c)}$$

where n depicts an integer, and f_{sn}, f_{yn} and f_{zn} specify the amplitudes of the surface loads in the s, y and z directions, respectively. Under this loading condition, the solution of the ordinary differential equations specified in Equation 5 is:

$$\varphi_k = a_k \sin\left(\frac{n\pi z}{L}\right) \quad \text{with} \quad k = 1, 2, \cdots, K \qquad (12)$$

in which φ_k represents the amplitude function related to the k-th mode which is expressed in terms of an unknown constant a_k. The structural response is then fully determined once these constants are evaluated. In the proposed numerical solutions, a unit value has been assigned to integer n included in Equations 11 and 12.

The results from the three methods (considering representative deformations and stresses) perfectly match (Figure 3). The mode participations related to the proposed and two-step classical GBT provide identical values as reported in Table 1 (mode participation greater than 1% have been reported). The modes influencing the overall response are 1-8, whose mode participation is greater than 4%. In the case of the dynamic modes, the mode participations are slightly different, since they are a combination of rigid and local/distortional modes, as already pointed out in the previous section.

Based on these solutions, it can be concluded that the proposed approach provides identical results (in terms of member analysis) to the dynamic procedure and the two-step classical GBT, therefore simplifying the overall derivation of the orthogonal modes within the framework of the GBT. The modes obtained with the dynamic procedure are shown to be similar, even if not exactly identical, to those derived with the other two approaches. Despite this, the numerical cases considered in this paper highlighted how the dynamic modes are still useful, because capable of describing the structural response based on a limited number of modes, identical, at least for the cases considered here, to the number required in the other two methods.

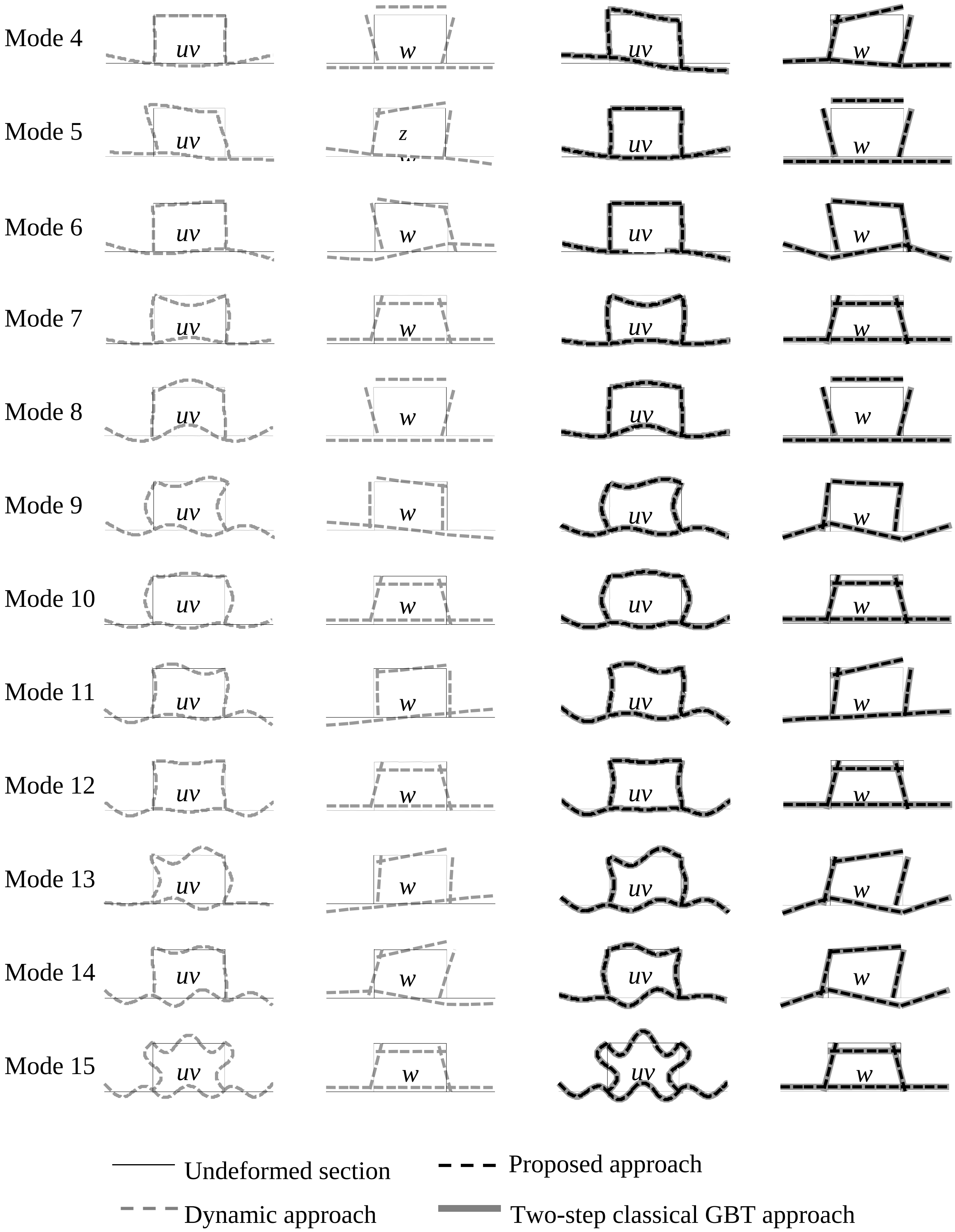


Figure 2: Comparisons between the conventional modes (excluding the rigid modes) calculated for the partially-closed section using the proposed method, the dynamic approach and the two-step classical GBT.

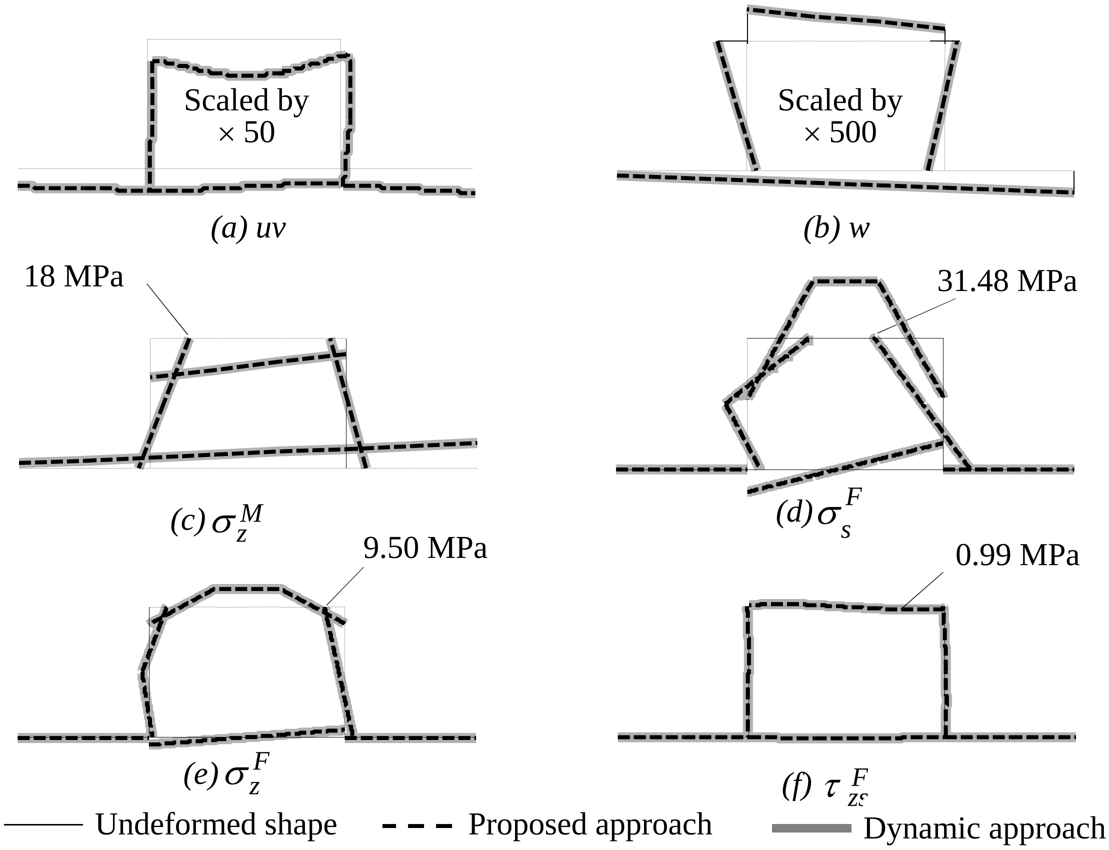


Figure 3: Stress distributions calculated for the partially-closed section

Table 1: Comparison between the values obtained for the unknown constants a_k calculated using the proposed approach, the dynamic procedure and the twp-step classical GBT approach for the partially-closed section

Mode number	Dynamic approach		Two-step classical GBT procedure		Proposed approach	
	Amplitude function a_k	Mode participation	Amplitude function a_k	Mode participation	Amplitude function a_k	Mode participation
1	0.0390	8.1	0.0286	4.8	0.0286	4.8
2	-0.1907	39.5	-0.1678	27.9	-0.1678	27.9
3	0.0004	0.1	-0.0444	7.4	-0.0444	7.4
4	0.0087	1.8	-0.0654	10.9	-0.0654	10.9
5	0.0291	6.0	-0.0398	6.6	-0.0398	6.6
6	-0.0498	10.3	0.0542	9.0	0.0542	9.0
7	0.0891	18.5	0.1101	18.3	0.1101	18.3
8	-0.0334	6.9	-0.0465	7.7	-0.0465	7.7
9	0.0114	2.4	-0.0155	2.6	-0.0155	2.6
10	0.0138	2.9	-0.0146	2.4	-0.0146	2.4
11	-0.0041	0.8	0.0033	0.5	0.0033	0.5
12	0.0084	1.7	0.0066	1.1	0.0066	1.1

4 CONCLUSIONS

This paper proposes the expression of a new functional for the derivation of the conventional modes in the framework of the GBT analysis. The proposed approach can be easily implemented for general cross-sections, including open, closed and partially-closed geometries. Its main advantage relies on its ability to determine an orthogonal set of conventional modes of any sections by means of a simple one-step numerical procedure.

The consistency of the proposed method has been discussed by means of a numerical example carried out on a partially-closed one. It has been shown that the conventional modes derived with the proposed (one-step) approach are identical to those determined with the classical two-step procedure. The modes obtained with the dynamic approach produce similar results, even if not exactly identical. The differences are due to the fact that the dynamic deformation modes may be combined with one or more rigid modes, which is not the case when the modes are determined with the other two approaches. It is also worth pointing out that the conventional GBT procedure requires the second diagonalisation step to produce non-local type modes with a physical meaning. Despite this, the numerical values calculated from the member analysis yield identical solutions, verified in terms of deformations and stresses, and are independent from the approach used to derive the conventional deformations modes.

The ability of these methods to derive an orthogonal set of modes within the GBT framework is useful because enabling to distinguish between distortional, local or pure torsional modes, and providing insight into how these influence the overall response of a thin-walled member from a structural viewpoint.

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